NanoWorld Journal

https://doi.org/10.17756/nwj.2023-s4-045

Research Article

Open Access

Post-buckling Analysis of Circular Plates Using Finite Element Method and Intuitive Formulation

Venkata Ramana Reddy Devarapalli[1,2*], Tholasi Balaram Padal Korabu[1], Rajesh Kumar Gupta[3] and Jagadish Babu Gunda[3]

[1]*Andhra University College of Engineering, Visakhapatnam, Andhra Pradesh, India*

[2]*Department of Mechanical Engineering, Vardhman College of Engineering, Hyderabad, Telangana, India*

[3]*Advanced Systems Laboratory, DRDO, Hyderabad, Telangana, India*

***Correspondence to:**
Venkata Ramana Reddy Devarapalli
Andhra University College of Engineering,
Visakhapatnam, Andhra Pradesh, India.

Department of Mechanical Engineering,
Vardhman College of Engineering, Hyderabad,
Telangana, India.
E-mail: ramana.vardhaman@gmail.com

Received: September 15, 2023
Accepted: November 23, 2023
Published: November 28, 2023

Citation: Devarapalli VRR, Korabu TBP, Gupta RK, Gunda JB. 2023. Post-buckling Analysis of Circular Plates Using Finite Element Method and Intuitive Formulation. *NanoWorld J* 9(S4): S262-S266.

Published by United Scientific Group

Abstract

This work applied an iterative Finite Element Method (FEM) and an intuitive formulation to the post-buckling analysis of isotropic circular plates. A straightforward iterative finite element formulation has been employed to take into account geometric non-linearity of the von-Karman type. The numerical value for the circular plate with immovable end considering clamped and hinged end boundary conditions is obtained and compared using a standard available FEM formulation and accuracy of Intuitive formulation is discussed for immovable ends of circular plate based on FEM numerical results. The plate produced radial stress because of moderately huge deflection is evaluated by using the converged FEM numerical results and radial tension variation across the radius of the circular plate with various boundary conditions is briefly summarized for various plate boundary conditions considered.

Keywords

Circular plates, Finite element formulation, Intuitive formulation, Post-buckling, Radial tension

Nomenclature

a = the circular plate's radius; c = the circular plate's central deflection; C = the plate's axial stiffness (Eh/(1-v2)); D = the plate's flexural rigidity ((Eh3)/12(1-v2)); Young's modulus = E; [ge] = matrix of element geometric stiffness; [G] = geometric stiffness matrix assembly; h = the plate's thickness; [KNL] = Assembly of the nonlinear stiffness matrix; Tr = radial tensile load per unit circumference; T = circumferential tensile stress per unit circumference.; Critical radial load = N_(r_cr); r = radial coordinate; U = energy of strain; W = work completed; generalized displacements = 1 x 8; δ = eigen vector; $\in_{è}, \in$ = radial and circumferential strains; λ_r = non-dimensional radial tensile load parameter = $\frac{T_r a^2}{D}$; λ_θ = non-dimensional circumferential tensile load parameter = $\frac{T_\theta a^2}{D}$; U = strain energy; u = axial displacement; w = lateral displacement; ν = Poisson ratio; α = coefficient of linear thermal expansion.

Introduction

Since Berger's well-known work in 1955, several researchers have examined the post-buckling examination of thin circular plates exposed to von-Karman strain-displacement relations. Bergers' [1] estimate involves ignoring the strain energy components corresponding to the plate middle surface's second invariant of strains. In the literature, applications for the related problems where the plate's edges are constrained against in-plane movements, Bergers approximation demonstrates that proper solutions are obtained. A modified energy expression created by Banerjee and Datta [2] results in decoupled huge deflection plate equations (similar to Berger's method).

Thermal post-buckling analysis of uniform, isotropic, thin, and shear flexible columns is described by Gupta et al. [3] utilizing a precise finite element formulation and a significantly simpler intuitive technique. Jones et al. [4] recommended employing a perturbation technique in situations when Berger's approach might not be accurate enough. Berger's method's accuracy in the situation of mechanical loading is correlated with the specific type of boundary condition given, according to Nowinski and Ohnabe [5]. Naidu et al. [6] investigates the post-buckling behavior of circular plates supported by a somewhat elastic axi-symmetric base. The nonlinear vibrations of tapered circular plates with edges that were elastically constrained against rotation were studied by Raju and Rao [7]. The post-buckling behavior of circular plates was studied by Raju and Rao [8-11]. According to Tauchert [12], the results of earlier studies are unquestionably relevant to both mechanical loads and thermal deflections. Once more, Ramaraju and Gundabathula [13], examine the post-buckling analysis of circular plates using an intuitive construction. Numerous studies [14-17] use finite elements and logical formulations to study vibration and post-buckling behavior. Rao and Varma [18] predicted the post-buckling load of circular plates with immovable ends using a simple notion. The authors assumed a set of permissible lateral displacement variation and then evaluated radial tensile load parameters analytically.

In thin circular plates, nanomaterials can be used to make the plates. There is a lot of scope in the nano material to include for studies. New perspectives in the domains of science and engineering have been opened up by the production of nanofibers from polymers that are natural or synthetic, metallic substances, semiconductors, composite materials, and carbon-based materials [19].

The post-buckling analysis of thin circular plates was investigated in this work utilizing a rigorous iterative finite element formulation with axially moveable and immovable ends. Intuitive formulation also makes use of the eigenvector derived via rigorous finite element formulation. By disregarding and using Berger's assumption, the tension parameter is assessed, and its effect on the projection of the post-buckling envelope of analytical results is briefly looked at. In addition to presenting radial and circumferential load fluctuations, the accuracy of numerical results derived utilizing Intuitive formulation is briefly explored in this paper.

Experimentation

Finite element formulation

The circular plate with small strains and large rotations can have the von-Karman strain-displacement relations [20] for axi-symmetric deformation.

$$\varepsilon_r = \frac{du}{dr} + \frac{1}{2}\left(\frac{dw}{dr}\right)^2 \tag{1}$$

$$\varepsilon_\theta = \frac{u}{r} \tag{2}$$

$$\psi_r = -\frac{d^2 w}{dr^2} \tag{3}$$

$$\psi_\theta = -\frac{1}{r}\frac{d^2 w}{dr^2} \tag{4}$$

The annular plate element's strain energy (U) for isotropic and elastic material characteristics may be represented as:

$$U = \frac{1}{2}\int_0^{2\pi}\int_{r_1}^{r_2}\left\{C\left[\varepsilon_r^2 + \varepsilon_\theta^2 + 2\nu\varepsilon_r\varepsilon_\theta\right] + D\left[\psi_r^2 + \psi_\theta^2 + 2\nu\psi_r\psi_\theta\right]\right\}drd\theta \tag{5}$$

Where r_1 and r_2 are the internal and exterior radii of the annular plate member.

The non-linear stiffness matrix is constructed in terms of u, u', and w' and w'' using the similar approach described in Rao and Raju [14-16].

$$[k]_{NL} = \int_0^{2\pi}\int_{r_1}^{r_2}\begin{bmatrix} \frac{C}{r^2} & \frac{\nu C}{r} & \frac{\nu w' C}{2r} & 0 \\ \frac{\nu C}{r} & C & \frac{C w'}{2} & 0 \\ \frac{\nu w' C}{2r} & \frac{C w'}{2} & \frac{D}{r^2} + \frac{C}{2}\left(u' + \frac{\nu u}{r} + w'^2\right) & \frac{\nu D}{r} \\ 0 & 0 & \frac{\nu D}{r} & D \end{bmatrix} drd\theta \tag{6}$$

For u and w over the element, cubic displacement distributions are assumed.

$$u = \alpha_1 + \alpha_2 r + \alpha_3 r^2 + \alpha_4 r^3 \tag{7}$$

$$w = \alpha_5 + \alpha_6 r + \alpha_7 r^2 + \alpha_8 r^3 \tag{8}$$

The element geometric stiffness matrix, written as, is computed using the work (W) that the external force has done.

$$W = \frac{1}{2}N_r\int_0^{2\pi}\int_{r_1}^{r_2}\left(\frac{\partial W}{\partial r}\right)^2 rdrd\theta \tag{9}$$

Where, W is the radial equivalent of the uniform compressive force per unit length. The matrix equation describing the plate's post-buckling behavior is found to be using the conventional constructing procedure.

$$[K_{NL}]\{\delta\} + \lambda[G]\{\delta\} = 0 \tag{10}$$

Equation 10 may be solved using a common approach [15] to extract the eigenvalues and eigenvectors.

The numerical values obtained using the conventional FEM technique mentioned in applying a least-squares curve fit of the proportion of post-buckling load variable to linear buckling load parameter, as shown below [7, 8].

$$\frac{\lambda_{NL}}{\lambda_L} = 1 + a_0\left(\frac{c}{h}\right)^2 + a_1\left(\frac{c}{h}\right)^4 \tag{11}$$

$$\frac{\lambda_{NL}}{\lambda_L} = 1 + a_2\left(\frac{c}{h}\right)^2 \tag{12}$$

Intuitional formula

The definition of the term "radial and circumferential tension developed per unit length of circumference" is given by:

$$T_r = \frac{Eh}{\left(1-v^2\right)}\int_0^a \left(\varepsilon_r + v\varepsilon_\theta\right)dr \tag{13}$$

$$T_\theta = \frac{Eh}{\left(1-v^2\right)}\int_0^a \left(v\varepsilon_r + \varepsilon_\theta\right)dr \tag{14}$$

This study determines the tension created in the circular plate and predicts the post-buckling behavior of circular plates with adjustable and immovable plate edges. If r is a non-zero value and the second invariant of strain is zero, then:

$$\varepsilon_\theta = 0 \tag{15}$$

When applying Berger's approximation, the following condition must be satisfied:

$$\varepsilon_\theta = 0 \tag{16}$$

The formulas for radial and tangential tension per unit length of circumference are as follows, using the circumstances in the equation above:

$$T_r = \frac{Eh}{\left(1-v^2\right)}\int_0^a \left(\varepsilon_r\right)dr \tag{17}$$

$$T_\theta = \frac{Eh}{\left(1-v^2\right)}\int_0^a \left(v\varepsilon_r\right)dr \tag{18}$$

Figure 1 and figure 2 demonstrate the fluctuation of these parameters in non-dimensional form for moveable and immovable end boundary conditions, respectively.

Once T_r and T_θ are the non-dimensional tension parameters are assessed using equation 13 and equation 14 or equation 17 and equation 18, which apply the Berger's assumption. $\left(\lambda_r = \frac{T_r a^2}{D}\right)$ and $\left(\lambda_\theta = \frac{T_\theta a^2}{D}\right)$ may be analyzed, and circular plate post-buckling analysis can be represented as:

$$\lambda_{NL} = \lambda_L + \lambda_r \tag{19}$$

The above equation may be rearranged as follows:

$$\frac{\lambda_{NL}}{\lambda_L} = 1 + \frac{\lambda_r}{\lambda_L} \tag{20}$$

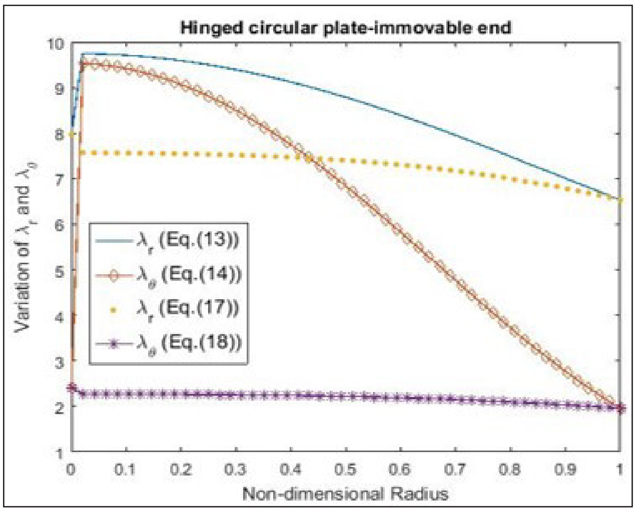

Figure 1: Variation of tension parameters λ_r and λ_θ for hinged and clamped circular plate with immovable end.

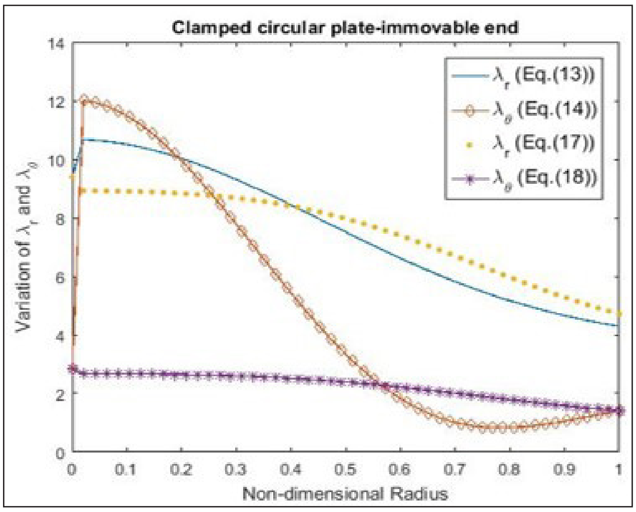

Figure 2: Variation of tension parameters λ_r and λ_θ for hinged and clamped circular plate with axially movable end.

Leaving out the numerical value of a_2 from the least square fitting equation, the post-buckling phenomena may be described as follows:

$$\frac{\lambda_{NL}}{\lambda_L} = 1 + b_I\left(\frac{c}{h}\right)^2 \tag{21}$$

It should be noticed that the b_I in equation 21 forecasts the coefficient terms pertaining to the order of the variables.

$$\left(\frac{c}{m}\right)^2$$

By equating the preceding equation 20 and equation 21, the tension approximation from the intuitive formulations may be used to get the coefficient b_I, which is expressed as:

$$b_I = \frac{\lambda_r}{\lambda_L}\frac{1}{\left(\frac{c}{h}\right)^2} \tag{22}$$

Circular plate equations

FEM vs analytical formulation for clamped circular plates' post-buckling behavior

$$w = \frac{b}{2}\left(1 + \cos\frac{\pi x}{L}\right) \tag{23}$$

$$u = \frac{1}{16}\frac{b^2\pi}{L}\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi x}{L}\right) \tag{24}$$

$$T_r = \frac{12D}{Lh^2}\int_0^L\left(u^1 + \frac{w^{1^2}}{2} + \frac{vu}{x}\right)dr \tag{25}$$

$$\lambda_T = \frac{T_r L^2}{D} = 7.90319440\frac{d^2}{h^2} \tag{26}$$

For Clamped circular plate

$$\lambda_L = 14.6824$$

$$\frac{P_{NL}}{P_L} = 1 + \frac{\lambda_T}{\lambda_L} = 1 + \frac{7.90319440}{14.6824}\frac{b^2}{h^2} = 1 + 0.538292\frac{b^2}{h^2} \tag{27}$$

For Hinged circular plate

$$\lambda_L = 4.1978$$

$$\frac{P_{NL}}{P_L} = 1 + \frac{\lambda_T}{\lambda_L} = 1 + \frac{7.90319440}{4.1978}\frac{b^2}{h^2} = 1 + 1.88275\frac{b^2}{h^2} \tag{28}$$

Results and Discussion

Table 1 displays the boundary conditions that were taken into consideration for the post-buckling analysis of a circular plate with an immovable edge. The post-buckling analysis results are reported in equation 11 and equation 12 using a formalized finite element method, and the calculated values of a_0, a_1, and a_2 are summarized in table 1 for each of the boundary conditions examined in this work.

The numerical outcomes of a postbuckling study of circular plates with immovable end conditions are shown in table 1 using rigorous FEM. The same findings are reported in this study to compare the accuracy of intuitive formulation with those found in the open literature.

The Intuitive formulation's equaiton 22 is used to forecast the post-buckling analysis of a circular plate, and equation 13 or equation 17 are used to obtain the radial tension parameter (r). Circular plates' post-buckling behavior when they have fixed ends when different tension estimations were used in the current experiment is shown in the accompanying table. For axially immovable ends, the average or integrated tension method overpredicts the coefficient a_1, but implementing Berger's assumption when assessing tension parameter leads to underprediction of the coefficient a_1. While the findings from the central node tension value for the hinged case are typically accurate, they show a minor divergence for the clamped plate with immovable edge. This method does not seem to produce a good estimate for the movable ends of the plate since intuitive formulation yields substantial variance.

- In the reference by Gupta et al. [3], numerical findings for nonlinear frequency are compared with linear change of u and cubic variation of w.
- As previously reported by Gupta et al. [3], cubic variation of u and w for beams with axially immovable end conditions demonstrates that At every node, the tension (membrane stretching force) created is zero of the beam's FEM idealization..
- In the current study, intuitive formulation results in pretty excellent approximation for immovable edge conditions of the plate, however it does not result in a decent approximation for moveable ends.
- It is possible that, when using an intuitive formulation, the influence of field inconsistency leads to inaccurate results in the post-buckling analysis of circular plates with moving edges. This effect may be further explored. The current work may serve as a motivator for determining the applicability of intuitive formulations for circular plates with movable ends. The same is being pursued by the authors and accuracy of such analytical methods will be discussed in future works planned.
- When equation 13 multiplied for $(1-\nu^2)$, results obtained from the present FEM based intuitive formulation (Table 2) shows an excellent agreement with the results obtained from rigorous finite element formulation for circular plates with immovable ends (Table 3) without invoking any further assumption on equation 13 in contrast to the previous analytical intuitive formulations reported on this problem of immovable ends.
- When equation 13 multiplied for $(1-\nu^2)$, conclusions from the current FEM based Intuitive formulation (Table 4) shows slight improvement for movable ends compared to rigorous finite element formulation results. However, as discussed above, field consistency of axial displacement filed effects will be investigated and discussed in future works planned.

Conclusion

Using a stringent finite element approach, the buckled post behavior of circular plates with clamps and hinges with

Table 1: Boundary requirements for the circular plate with axial limitations were taken into consideration (u(0) = u(a) = 0).

S. No.	Boundary configuration	Boundary conditions on w
1	Clamped	$w'(0) = w(a) = w'(a) = 0$
2	Hinged	$w'(0) = w(a) = 0$

Table 2: Boundary specifications for a circular plate with a changeable edge.

S. No.	Boundary configuration	Boundary conditions
1	Clamped	$u(0) = w'(0) = w(a) = w'(a) = 0$
2	Hinged	$u(0) = w'(0) = w(a) = 0$

Table 3: Findings of a thorough finite element formulation-based post-buckling study.

A circular plate having a fixed end			
	a0	a1	a2
Hinged	1.82201	0.059731	1.86674
Clamped	0.524179	0.004345	0.52883

Table 4: Results of post-buckling analysis utilizing intuitive formulation: Analytical vs FEM.

	Hinged	Clamped
λ_r (*FEM*)	7.8021	7.7523
λ_L (*FEM*)	4.1978	14.6824
λ_r (*Analytical*)	7.9034	7.9034
b_I (*FEM*)	1.8586	0.5280
b_I (*Intuitive*)	1.88275	0.5383

fixed end conditions is once more examined. The strict finite element formulation's eigenvector is then used in the intuitive formulation to forecast the post-buckling behavior of hinged and clamped circular plates under circumstances involving immovable ends. The accuracy of intuitive formulations based on FEM numerical findings for circular plates with immovable end boundary constraints is explored. The intuitive technique yields reasonable approximations for circular plates with immovable ends, but the results for moving ends are erroneous. The eigenvector produced by a stringent for clipped and hinged boundary conditions, a finite element formulation is moveable and immovable end conditions is used to demonstrate how radial and circumferential tensile load characteristics may change. The outcome of the present study is a major motivating factor to pursue further in improving the accuracy of the existing analytical intuitive formulations relevant to circular plates with immovable edges and the same is being pursued rigorously with isotropic as well as orthotropic circular plates which will be reported in the immediate works planned by the authors.

Acknowledgements

None.

Conflict of Interest

None.

References

1. Berger HM. 1955. A new approach to the analysis of large deflections of plates. *J Appl Mech* 22(4): 465-472. https://doi.org/10.1115/1.4011138
2. Banerjee B, Datta S. 1981. A new approach to an analysis of large deflections of thin elastic plates. *Int J Non Linear Mech* 16(1): 47-52. https://doi.org/10.1016/0020-7462(81)90031-7
3. Gupta RK, Babu GJ, Janardhan GR, Rao GV. 2009. Relatively simple finite element formulation for the large amplitude free vibrations of uniform beams. *Finite Elem Anal Des* 45(10): 624-631. https://doi.org/10.1016/j.finel.2009.04.001
4. Jones R, Mazumdar J, Cheung YK. 1980. Vibration and buckling of plates at elevated temperatures. *Int J Solids Struct* 16(1): 61-70. https://doi.org/10.1016/0020-7683(80)90095-5
5. Nowinski JL, Ohnabe H. 1972. On certain inconsistencies in Berger equations for large deflections of plastic plates. *Int J Mech Sci* 14(3): 165-170. https://doi.org/10.1016/0020-7403(72)90073-2
6. Naidu NR, Raju KK, Rao GV. 1993. Post-buckling behaviour of circular plates resting on an axi-symmetric elastic partial foundation under uniform compressive loads. *Comput Struct* 46(1): 187-190. https://doi.org/10.1016/0045-7949(93)90179-H
7. Raju KK, Rao GV. 1979. Nonlinear vibrations of tapered circular plates elastically restrained against rotation at the edges. *Nuclear Eng Des* 51(3): 417-421. https://doi.org/10.1016/0029-5493(79)90129-8
8. Raju KK, Rao GV. 1984. Thermal post-buckling of circular plates. *Comput Struct* 18(6): 1179-1182. https://doi.org/10.1016/0045-7949(84)90162-7
9. Raju KK, Rao GV. 1986. Post-buckling behaviour of moderately thick circular plates with edges elastically restrained against bending rotation. *Comput Struct* 23(3): 457-458. https://doi.org/10.1016/0045-7949(86)90236-1
10. Raju KK, Naidu NR, Rao GV. 1996. Thermal buckling of circular plates with localized axisymmetric damages. *Comput Struct* 60(6): 1105-1109. https://doi.org/10.1016/0045-7949(96)00016-8
11. Raju KK, Rao GV. 1996. Thermal post-buckling of linearly tapered moderately thick isotropic circular plates. *Comput Struct* 58(3): 655-658. https://doi.org/10.1016/0045-7949(95)00150-F
12. Tauchert TR. 1986. Thermal Stresses in Plates. In Hetnarski RB (ed) Statical Problems in Thermal Stresses I. Elsevier.
13. Ramaraju RV, Gundabathula VR. 2009. Reinvestigation of intuitive approach for thermal postbuckling of circular plates. *AIAA J* 47(10): 2493-2495. https://doi.org/10.2514/1.43034
14. Rao GV, Raju KK. 1983. A reinvestigation of post-buckling behaviour of elastic circular plates using a simple finite element formulation. *Comput Struct* 17(2): 233-235. https://doi.org/10.1016/0045-7949(83)90011-1
15. Rao GV, Raju KK. 1984. Optimization of cylindrically orthotropic circular plates including geometric non-linearity with a constraint on the fundamental frequency. *Comput Struct* 18(2): 301-305. https://doi.org/10.1016/0045-7949(84)90128-7
16. Rao GV, Raju KK. 1986. A study of various effects on the stability of circular plates. *Comput Struct* 24(1): 39-45. https://doi.org/10.1016/0045-7949(86)90333-0
17. Rao GV, Raju KK, Naidu NR. 1992. Post-buckling behaviour of circular plates with elastically restrained edges subjected to thermal loads. *Comput Struct* 45(1): 209-210. https://doi.org/10.1016/0045-7949(92)90358-7
18. Rao GV, Varma RRG. 2007. Simple formulation to predict the thermal post-buckling of circular plate, *AIAA J* 45(7): 1784-1786.
19. Rao HJ, Ramulu PJ, Vardhan MV, Chandramouli CH. 2016. Failure prediction in fiber metal laminates for next generation aero materials. *IOP Conf Ser Mater Sci Eng* 149(1): 012102. https://doi.org/10.1088/1757-899X/149/1/012102
20. Timoshenko S, Woinowsky-Krieger S. 1959. Theory of Plates and Shells. McGraw-Hill, New York.