Nano World Journal

https://doi.org/10.17756/nwj.2023-s4-054

Research Article

Open Access

A Study on Critical Speed Analysis of Rotating Shaft

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Received: September 15, 2023
Accepted: November 24, 2023
Published: November 29, 2023

Citation: Jena SP, Jettigari SR, Goud K, Motapothula BK, Pandya K. 2023. A Study on Critical Speed Analysis of Rotating Shaft. *NanoWorld J* 9(S4): S320-S324.

Published by United Scientific Group

Abstract

The vibration analysis of rotating structure is an emerging topic in the era of structural dynamics. When the rotational speed of the shaft is approaching towards the natural frequency of the structure, there may be the occurrence of resonance which leads to vibrate vigorously. The present work is based on the transverse vibration analysis of a rotating shaft. The natural frequencies and mode shapes of the rotating shaft are determined at various modes. The critical speed and resonance analysis on the response of vibrated rotating shaft are also discussed in this work. The mathematical model for the rotating shaft has been considered first to obtain the generalized equation of the proposed structure in theoretical way. Later, the Finite Element Analysis (FEA) was carried out to validate the proposed theoretical method. Numerical analysis has been carried out to compare the exactness of the two methods and found to be very fine convergent.

Keywords

Resonance, Critical speed, Shaft, Natural frequency

Introduction

Shaft is one of the major mechanical components. It is used to transmit the power from source to the system. It is mostly used to transmit the power in automobiles, turbines, compressors, and power houses. Now a day's shaft is made up of different materials. The materials like structural steel, aluminum, aluminum alloy, carbon steel, titanium, bronze, and nickel-based alloys. Different materials possess different characteristics that may help to sustain for a long time. But one of the major problems in rotating shaft is vibration. Vibration has a lot of disadvantages in machines. It may lead to loss of joints, noise, imbalance in rotating parts and damages the machine parts.

Vibration may cause due to different factors like imbalance in rotating parts, misalignment between the component and shaft, wear and tear of shaft, improper bearing performance and resonance. When the machine part oscillating with its natural frequency and approaches the resonance at certain speed that speed is called critical speed or whirling speed then shaft will vibrate. The main aim is to detect the occurrence of resonance and conditions to avoid, when the shaft is made up of aluminum alloy and simply supported at both ends. The analysis can be done theoretically and simulation [1].

Chinnapandiana et al. [2] have investigated the critical speed of rotating toy shaft which is less in dimensions. Shaft is made up of mild steel which is 5 mm diameter and 880 mm length supported at both ends with the bearings. One side is connected bearing with the motor and other side connected to the fixed bearing. Non-contact sensors are chosen and placed at the middle to identify the deflection at applied speed. Results are validated theoretically and with the help of non-contact sensors. Tripathi and Jain [3] studied the critical speed and natural frequency of rotating shaft by varying the diameter and the shaft materials under constant angular velocity 3663 rad/s chosen different diameter profiled shaft between 60 - 100 mm. Predicted natural frequency of different shaft diameters and different lengths by using Experimental and ANSYS Workbench. Baig et al. [1] studied the critical speed of an induction motor, when the rotor is mounted at the center. Chosen the steel shaft acting the radial load at one end and rotor weight at the center the design is made on the CATIA, and it imported into the ANSYS for simulation. Chosen the theoretical and simulation approaches to validate the results.

Desai et al. [4] studied the critical speed of shafts of various diameters theoretically and experimentally and also evaluated the self-excited motion by gradually increasing speed of the shaft, in experimental approaches chosen the two different shafts for two different conditions with rotor and without rotor by simply supported with the fixed bearings. Raghunandan et al. [5] also analyzed the critical speed of rotating shaft. The shaft is made up of structural steel and evaluated by applying the different boundary conditions i.e., simply supported and cantilever. Results are validated by the theoretical, experimental and FEA by plotting the graph between the frequency against rotational velocity i.e., which was called as Campbell graph. To understand the dynamic behavior of rotating shaft, Firouzi and Ghassemi [6] have investigated the whirling speed of ship propeller shaft and evaluated the effect of diameter and torsional stiffness of the shaft. The shaft is used to transmit the power from source to the transmission system with gear box. They have also carried out the analysis of whirling speed of shaft at different gear conditions and evaluated by theoretical and FEA.

Yang et al. [7] developed the analytical solution for whirling shaft at different modes and with the different end conditions. The shaft is modeled by considering Rayleigh beam model and the mechanism of gyroscopic effects. They have chosen the six boundary conditions to determine whirling speed of the shaft. Ashok et al. [8] explored the critical speed of rotating shaft with the different diameters at different boundary conditions by taking it account the structural steel shaft simply supported. The results were also verified by the numerical, experimental and FEA approaches. Jadhav et al. [9] investigated the critical speed of shaft carrying the single rotor. They have chosen the different diameters of shaft with same length and same material and founded the natural frequency by following the Dunkerley's formula and determined the critical speed for different diameters. To understand the effect of diameter for the whirling speed and natural frequency they have plotted the two graphs, critical speed vs mass and natural frequency vs mass at different diameters. Geonea et al. [10] investigated the critical speed of right shafts when it simply supported by the two bearings at both ends and they have chosen the shaft which has heavy disk at the middle. They have made a mathematical approach to find out the critical speed and performed simulation in the ADAMS multibody software to find the elastic deformation of shaft during rotating and made the experimental to investigate the critical speed. The results which are obtained by the analytical are verified by the experimental way.

Ghoniem et al. [11] founded the critical speed of shaft disc system has been calculated by different methods when the shaft is simply supported at its ends, and it consists of two symmetric disc certain distance from the centre of the shaft. The natural frequency of shaft at different modes are calculated using modal analysis to validate the results. Zhao et al. [12] has investigated free vibration behaviors of a functionally graded disk-shaft rotor system reinforced with graphene nanoplatelet resting on elastic supports. They are assumed material properties are varied by shaft radius and directions. The equations are derived for theoretical analysis from the Lagrange and Timoshenko beam theory. The results are validated by experimental and FEA analysis. Belhadj et al. [13] has analyzed the vibration behavior of nanoscale rotating shaft based single-walled carbon nanotube. They have derived the equations by different approaches. The Bernoulli beam equation is used to investigate the dynamic behavior of nano rotor, the governing equations and boundary conditions are derived by using Hamiton's principle all these equations are validated by the generalized differential quadrature methods. Bağdatlı [14] has investigated the non-linear vibrations of nanobeams by varying boundary conditions. The equation for non-linear vibration has derived by considering the stretching of the neutral axis. They have derived exact solutions for the mode shapes and frequencies by varying the boundary conditions. Drawn a frequency-response curve based on the results. Ghodousi et al. [15] has analyzed the non-linear vibrations and stability of rotating asymmetrical nano-shafts by considering the surface effects. They have used two theories for analysis. The theories are surface stress tensor and surface elastic theory. They have obtained the governing nonlinear equations of motion with the aid of variational approach. The results are validated with theories.

When the shaft rotates at certain speed with its natural frequency, vibration will occur which has many disadvantages. There is chance of resonance when the natural frequency coincides with the forcing frequency. In this work, the key objectives are to find the natural frequency of the rotating shaft at different speed to avoid the critical speed and resonance of the rotating structure. Here, an aluminum alloy shaft has been considered for the analysis. The present analysis includes both theoretical and FEA methods with different numerical examples.

Experimentation

Theoretical approach: Natural frequency equation at different modes

Free vibration equation for transverse vibration of shaft.

$$\mathrm{EI}\frac{\mathrm{d}^4 y}{\mathrm{d}x^4} + \frac{m\mathrm{d}^2 y}{\mathrm{d}t^2} = 0$$

$$\text{let } y(x,t) = \phi(x)\sin\omega t$$

$$\frac{d^4\phi}{dx^4} - \lambda^4\phi = 0, \text{ where } \lambda^4 = \frac{m\omega^2}{EI}$$

$$\phi(x) = A_1\cosh\lambda x + A_2\sinh\lambda x + A_3\cos\lambda x + A_4\sin\lambda x$$

Where, A_1, A_2, A_3, and A_4 are constants of integration that have to be found from boundary conditions.

$$\mathrm{EI}\frac{d^2 y}{dx^2} = 0$$

$$\phi(x) = A_1 \cosh \lambda x + A_2 \sinh \lambda x - A_3 \cos \lambda x - A_4 \sin \lambda x$$

$$\phi''(x) = \lambda^2 A_1 \cosh \lambda x + A_2 \lambda^2 \sinh \lambda x - A_3 \lambda^2 \cos \lambda x - A_4 \lambda^2 \sin \lambda x$$

Boundary conditions for simply supported ends.

$$\text{At } x = 0,\ \phi(0) = 0 \text{ and } EI\frac{d^2\phi}{dx^2} = 0$$

$$\text{At } x = L,\ \phi(L) = 0 \text{ and } EI\frac{d^2\phi}{dx^2} = 0$$

First condition gives $A_1 + A_3 = 0$ and $A_1 + A_3 = 0$, which means $A_1 = A_3 = 0$.

$$\text{At } x = L,\ \phi(L) = A_2 \sin A\lambda L + A_4 \sin \lambda L = 0$$

$$\phi''(L) = A_2 \lambda^2 \sinh \lambda L - A_4 \lambda^2 \sin \lambda L = 0$$

Hence, we get:

$$A_2 \sinh \lambda L + A_4 \sin \lambda L = 0$$

$$A_2 \sinh \lambda L - A_4 \sin \lambda L = 0$$

Adding these two equations,

$$2A_2 \sinh \lambda L = 0$$

$$\sinh \lambda L = 0$$

$$\lambda L = n\pi, \qquad n = (1,2,3,.....)$$

$$\lambda = \frac{n\pi}{L}$$

$$\omega_n = \lambda^2 \sqrt{\frac{EI}{m}}$$

$$\omega_n = \left(\frac{n\pi}{L}\right)^2 \text{ x } \sqrt{\frac{EI}{m}}$$

$$f_n = \frac{\omega_n}{2\pi}$$

$$F_n = \frac{\pi}{2} \text{ x } n^2 \text{ x } \sqrt{\frac{gEI}{wL^4}}$$

Theoretical calculations

f_n is Natural frequency, Hz; n is Type of mode i.e., 1,2,3…; I is Moment of inertia; E = Modulus of elasticity, GPa; w is Weight of shaft; L = Length of shaft, m; g = Acceleration due gravity, m/s²; Material is Aluminum alloy; Length is 1 m; Diameter = 5 mm = 0.005 m.

$$A = \pi/4 \text{ x } D^2 = \pi/4 \text{ x } 0.005^2 = 1.96 \text{ x } 10^{-5} \text{m}^2$$

$$V = A \text{ x } L = 1.96 \text{ x } 10^{-5} \text{ x } 1 = 1.96 \text{ x } 10^{-5} \text{m}^3$$

$$I = \frac{\pi d^4}{64} = \frac{\pi}{64} \text{ x } (0.005)^4 = 3.06 \text{ x } 10^{-11} \text{m}^4$$

$$m = \rho \text{ x } V$$

$$m = 2770 \text{ x } 1.96 \text{ x } 10^{-5} \qquad \rho = 2770 \text{ kg/m}^3$$

$$m = 0.054292 \text{ kg}$$

$$\ddot{u} =$$

$$w = 0.054292 \text{ x } 9.81$$

$$w = 0.5327 \text{ N}$$

$$f_{n1} = \frac{\pi}{2} \text{ x } n^2 \text{ x } \sqrt{\frac{gEI}{wL^4}} \qquad n = 1$$

$$f_{n1} = \frac{\pi}{2} \text{ x } (1)^2 \text{ x } \sqrt{\frac{9.81 \text{ x } 7.1 \text{ x } 10^{10} \text{ x } 3.06 \text{ x } 10^{-11}}{0.5327 \text{ x } 1^4}}$$

$$f_{n1} = 9.93667 \text{ Hz}$$

$$f_{n2} = \frac{\pi}{2} \text{ x } (2)^2 \text{ x } \sqrt{\frac{9.81 \text{ x } 7.1 \text{ x } 10^{10} \text{ x } 3.06 \text{ x } 10^{-11}}{0.5327 \text{ x } 1^4}} \qquad n = 2$$

$$f_{n2} = 39.7468 \text{ Hz}$$

$$f_{n3} = \frac{\pi}{2} \text{ x } (3)^2 \text{ x } \sqrt{\frac{9.81 \text{ x } 7.1 \text{ x } 10^{10} \text{ x } 3.06 \text{ x } 10^{-11}}{0.5327 \text{ x } 1^4}} \qquad n = 3$$

$$f_{n3} = 89.43 \text{ Hz}$$

FEA analysis

To find the natural frequency of shaft on ANSYS workbench 2023 R1. First, we need to give the engineering data. After we need to go into the geometry to model the shaft. Draw the circle with 5 mm diameter and extrude it into 1000 mm. After finishing the geometry open the model, assign the material, generate a mesh with appropriate element size, apply the boundary conditions, set the modes in the analysis settings. Obtain the natural frequency by solving the deformation in the solution command. Here the results are obtained in the ANSYS Workbench 2023 R1.

Results and Discussion

- In the present study, the transverse vibration analysis of a rotating shaft has been carried (Table 1).
- An aluminum shaft is considered for the entire analysis.
- The theoretical along with the FEA techniques are carried out to find the natural frequency of the rotating shaft at different modes.

Table 1: Properties.

S. No.	Property	Value	Unit
1	Modulus of elasticity	71	GPa
2	Mass density	2770	Kg/m^3
3	Poisson's ratio	0.33	-
4	Ultimate tensile strength	310	MPa
5	Yield strength	276	MPa

Table 2: Theoretical values and FEA analysis values.

S. No.	Modes	Theoretical values (Hz)	FEA analysis (Hz)	% Error
1	First mode	9.93667	9.9361	0.0057
2	Second mode	39.7468	39.741	0.0171
3	Third mode	89.43	89.404	0.029

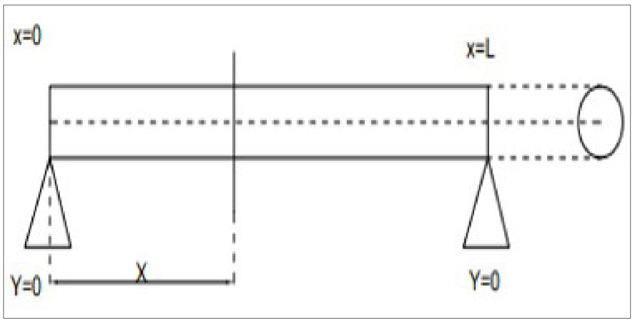

Figure 1: Shaft is simply supported at both ends.

- The results obtained from both methods are shown in table 2.
- Figure 1 presents shaft is simply supported at both ends, figure 2 presents shaft supported at both ends, figure 3 shaft rotating at different modes, figure 4 presents schematic diagram of the shaft at mode 1, figure 5 presents schematic diagram of the shaft at mode 2, and figure 6 presents schematic diagram of the shaft at mode 3.

$$\% \text{ of error} = \frac{\textit{theoretical value} - \textit{FEA analysis value}}{\textit{theroretical value}} \text{ x } 100$$

$$\% \text{ of error} = \frac{9.93667 - 9.9361}{9.93667} \text{ x } 100 = 0.0057$$

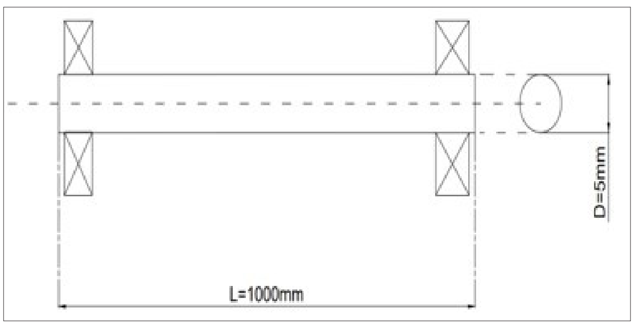

Figure 2: Shaft supported at both ends.

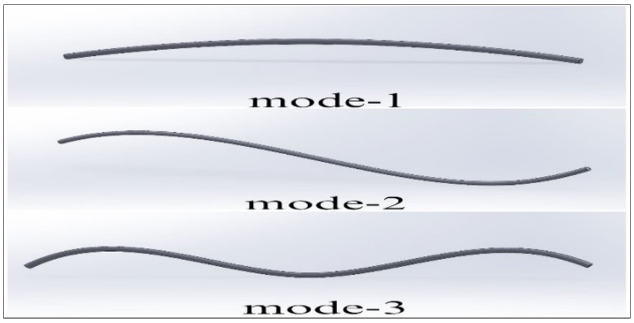

Figure 3: Shaft rotating at different modes.

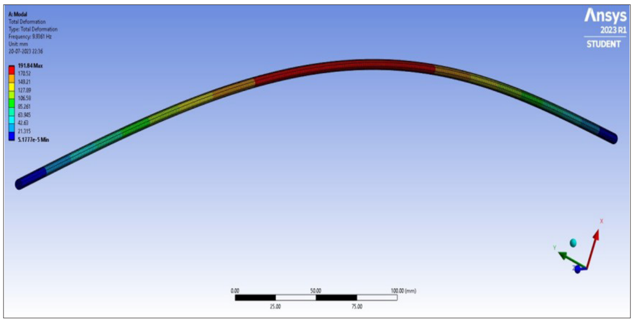

Figure 4: Schematic diagram of the shaft at mode 1.

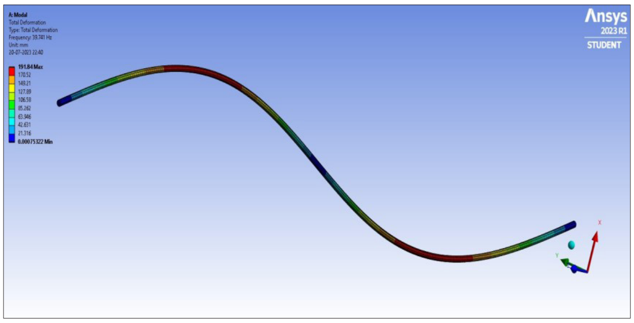

Figure 5: Schematic diagram of the shaft at mode 2.

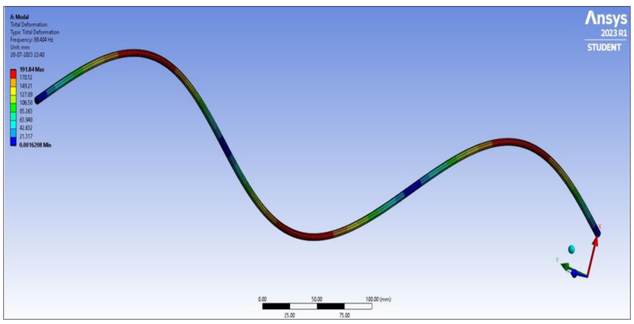

Figure 6: Schematic diagram of the shaft at mode 3.

$$\% \text{ of error} = \frac{39.7468 - 39.741}{39.7468} \text{ x } 100 = 0.0171$$

$$\% \text{ of error} = \frac{89.43 - 89.404}{89.43} \text{ x } 100 = 0.029$$

Conclusion

In this work it has been found that the natural frequency of aluminum alloy rotating shaft at different modes due to self we. The theoretical analysis has found using Euler's beam theory and FEA technique are carried out to validate the results for natural frequency at different modes. Both theoretical and numerical values are very close. The percentage of error is very less this technique is good one to analyze the critical speed of rotating shaft. Analyzing the critical speed engineering must ensure that design and operating speed of the shaft should be less than critical speed to avoid the vibration and enable the operating conditions should be safe and stable.

Acknowledgements

None.

Conflict of Interest

None.

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