Nano World Journal

https://doi.org/10.17756/nwj.2023-s1-052

Research Article

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Numerical Study on Natural Convection Inside Quadratic Enclosure with Uniform or Non-uniform Heating from Below

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Received: November 24, 2022
Accepted: April 14, 2023
Published: April 16, 2023

Citation: Mullick SH, Majumdar S, DasGupta D, Kundu PK. 2023. Numerical Study on Natural Convection Inside Quadratic Enclosure with Uniform or Non-uniform Heating from Below. *NanoWorld J* 9(S1): S266-S271.

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Abstract

Predicting thermal flow behaviors inside a quadrantal enclosure is the foremost impartial of the study with the non-uniform and uniform profiles of heat transfer through the bottom wall where the upright wall is sustained at comparatively low temperature and the curved wall is shielded. For understanding the buoyancy driven thermal flow behavior, vortex patterns, isotherms and Nu along the conduction wall are obtained under various Raleigh numbers ($10^4 \leq Ra \geq 10^6$). In this work, sinusoidal and constant temperature profiles have been applied from below. As Ra amplifies, the intensity of the buoyancy inside the enclosure rises, which results in an enhanced heat transit rate. Multiple circulations are found to form at high Ra in case of non-uniform bottom wall heating.

Keywords

Rayleigh number, Nusselt number, Stream functions, Isotherms, Air

Nomenclature

H is Height of the enclosure, m; k is Thermal conductivity, $Wm^{-1}K^{-1}$; P is Non-dimensional pressure component; U is Dimensionless velocity component along x; V is Dimensionless velocity component along y; α is Thermal diffusivity, m^2s^{-1}; T is Volumetric expansion coefficient, K^{-1}; T is Temperature of the fluid, K; c_p is Gravitational acceleration, ms^{-2}; c_p is Specific heat at constant pressure, $Jkg^{-1}K^{-1}$; θ Dimensionless temperature; Ra is Rayleigh number; Pr is Prandtl number; ρ is Nusselt number; ρ is Density of fluid, kgm^{-3}; v is Kinetic viscosity, m^2s^{-1}.

Introduction

Free convection inside cavity has been a more important research area through the last few periods for an extensive variety of industrial and domestic purposes such as boiler drum, cooling system of electronic devices, nuclear reactor core, material processing industries, etc. [1-16]. Sivasankaran and Bhuvaneswari [10] have surveyed free convection in a cavity having square shape and having sinusoidal heating from sides and by possession the horizontal walls shielded. They have perceived that local rate of heat transits is meaningfully prejudiced by sinusoidal profile of temperature for the side walls. In their research on natural convection inside square enclosures, Deng and Chang [17] study the influence of air density, aspect ratio, amplitude, and phase shift angle of sinusoidal profiles of temperature. They observed the greatest heat transfer rate when both sides have the same phase. Numerical predictions of free convection inside the enclosures are made by Mullick et al. [1] as top wall heating and keeping both the sides are

comparatively cold. They varied Ra, Pr, and heating frequency and observed that heat flow behaviors significantly alter with these parameters. However, more influences are found for Ra and heating frequency as compared to Pr. Few literatures are available in which conjugate convection has been thoroughly investigated using the vortex patterns, temperature distribution and Nu along the transference wall [18-20]. They have noticed that average Nu declines with the rise in wall thickness. Acharya and his co-researchers [21-24] have studied the natural convection phenomena using long, solid, or hollow horizontal cylinders with suspended air and non-Newtonian fluid to alter the thermal flow dynamic.

Based on the available literature it can be seen that the geometry and thermal condition of the walls play important roles for thermal flow alteration inside the cavity. There are no studies that have used stream function, isotherms, and Nu to critically analyze the thermal flow behaviour due to buoyancy force inside quadratic enclosures filled with air by introducing uniform and non-uniform heating from below. In this present study, the numerical simulation of thermal flow behaviour due to buoyancy force inside a quadratic enclosure has been thoroughly inspected using stream function, isotherms, and local Nu under consistently and uneven sinusoidal heating conditions from below.

Problem Definition and Mathematical Modeling

In this present enquiry, the upright wall is kept at a fairly low temperature and the bent wall is assumed to be zero heat loss in nature as shown in figure 1. The occupied fluid inside the enclosure is assumed to be air (Pr = 0.7) and Ra is varied from 10^4 to 10^6 where constant and sinusoidal heating are functional from the bottommost wall.

In order to simplify the study, the flow is implicit to be 2-D; the fluid is Newtonian in nature who's all the physical properties persist constant, apart from the density in the momentum equation that fluctuates according to Boussinesq approximation. Viscous dissipation and radiant heat transfer are supposed to be negligible [25].

Based on these assumptions, the following governing differential equations in dimensional forms are obtained.

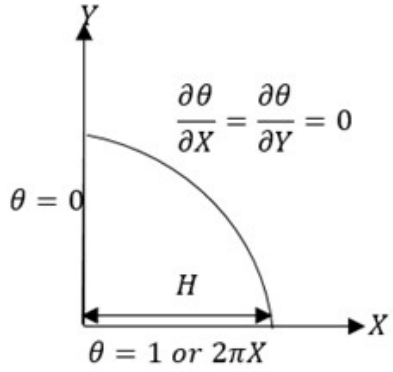

Figure 1: Schematic of problem domain.

$$\text{Continuity: } \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$\text{Momentum: } u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \quad (2)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial \rho}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+g\beta(T-T_o) \quad (3)$$

$$\text{Energy: } u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \quad (4)$$

Where, v and u stand for vertical and horizontal components of velocity, respectively. Density; kinematic viscosity; thermal diffusivity and pressure are represented, respectively as ρ, ν, α and p; T_o represents the top wall temperature. The following schemes are introduced to convert the governing equation in their corresponding non-dimensional forms.

$$X=\frac{x}{H};Y=\frac{y}{H};U=\frac{uH}{\alpha};V=\frac{vH}{\alpha};P=\frac{pH^2}{\alpha};\theta=\frac{T-T_o}{\Delta T}$$

The non-dimensional governing equations are embodied by equations 5 to 8.

$$\text{Continuity: } \frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad (5)$$

$$\text{Momentum: } U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \quad (6)$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ra\Pr\theta \quad (7)$$

$$\text{Energy: } U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2} \quad (8)$$

Where, Ra; Pr epitomize Rayleigh and Prandtl numbers, respectively as demarcated by $\Pr=\frac{\nu}{\alpha}$ and

$$Ra=\frac{\rho g\beta\Delta T H^3}{\mu\alpha}.$$

The boundary settings at different walls are prearranged below.

Vertical: U = V = θ = 0 for 0 ≤ Y ≤ 1 and X = 0;

Bottom: U = V = 0, θ = 1/sin (2πX) for 0 ≤ Y ≤ 1 and X = 0;

Curved wall: U = V = 0, $\frac{\partial\theta}{\partial X}=\frac{\partial\theta}{\partial Y}=0$

The governing differential equations (5 to 8) are solved utilizing ANSYS Fluent software through finite volume method. In momentum equations, speed and pressure are

handled using the SIMPLE algorithm [26-29]. The central difference and second-order upwind schemes are used for diffusive and convective terms, respectively. For uncertain terms, first-order implicit schemes are used whereas the buoyancy having y-momentum equation is exhibited by a user-defined function. Governing equations with nonlinear and coupled characteristics are solved using the Gauss-Seidel method. The residuals for convergence criteria of momentum, continuity, and energy are considered as 10^{-6}.

For growing an accurate solution of the existing work, first code authentication has been executed by comparing this with the work of Aydin and Yesiloz [20], and the results are described in figure 2. One can be clearly visible that the present work is in reasonable match with the past work indicating successful benchmarking of the code. The code validation is further carried out by determining and comparing the average heat transfer rate along the conduction wall in between the current code and previous work as detailed in table 1. It can be found from table 1 that the average heat transfer rate also matches closely with the existing work, successfully benchmarking the present code.

The numerical investigation has also been subjected to the grid independence test to confirm the generation of the correct solutions. For grid independency study, the parameters selected for the work are Ra = 10^5 and constant heating from below (θ = 1). The average Nu along the bottommost wall is estimated for each refinement of grids and the results of the consecutive grid refinements are compared to see the changes in the results as detailed in table 2.

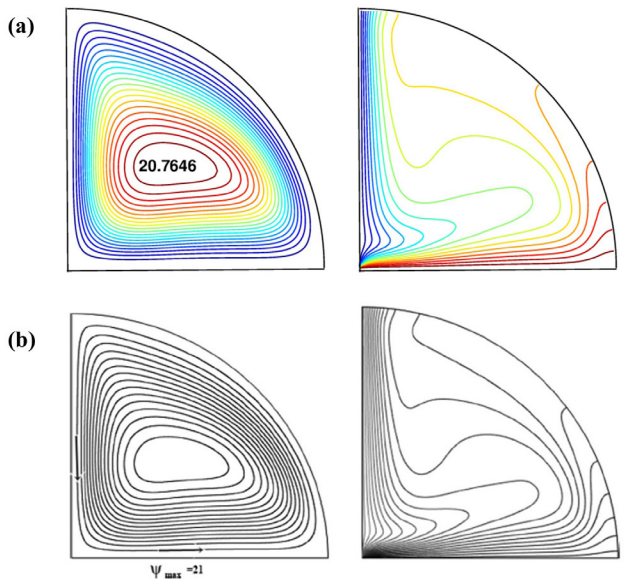

Figure 2: Contour plots **(a)** contemporary work and **(b)** work of Ayadin and Yesiloz [20].

Table 2: Error analysis for grid dependency test.

Sl. No.	Number of nodes	Number of elements	Avg-Nu	% in change of Avg-Nu
1	6144	**11684**	**2.61**	-
2	13672	**26442**	**2.40**	**11.9**
3	23971	**46732**	**2.405**	**0.3**

The number of nodes and elements used are chosen at 13672 and 26442, respectively, to prevent grid dependency as it can be noticed from table 2 that the difference in results is only 0.3% for subsequent refinement.

Results and Discussion

For the computation of the thermal flow behaviors, influence of Ra has been discussed with the help of contour plots of isotherms and stream function. Then local Nu along both bottom and vertical walls has been critically scrutinized in the following subsections.

Stream function and temperature distribution

Natural convection is a strong function of Ra and Pr. Keeping Pr constant at 0.7, variations of isotherm and stream function patterns under different Ra during uniform heating are displayed in figure 3. One can be perceived from figure 3 that the circulation intensity is a smaller amount at the lower values of Ra (Ra = 10^4) resulting in circular patterns of stream functions. This is attributable to the lower potency of buoyancy force that leads to conduction leading heat flow, which is also supported by the smooth natures of the isotherms. Furthermore, it is found that the circulation strength extremely augments with the upsurges in Ra because of significant augmentation of buoyancy force that prolongs the convection strength within the space as can be perceived in figure 3b and 3c. From the isotherm patterns, it is also noticed that these bend toward the vertical wall indicating significant transfer of heat from the hot lowermost wall to cold upright wall. High strength circulation roll is found nearer to the core that tends to zero gradually towards the wall. Attributable to no slip boundary condition at the wall.

Figure 4 represents the isotherm and stream function patterns under different Ra during non-uniform sinusoidal heating. Similar kind of variation has been noticed with Ra as in the cases of constant temperature heating. At higher values of Ra, multiple circulations are observed owing to boosted buoyancy force and the power of these circulation rolls improves with the rise in Ra (Figure 4b and 4c). Two thermal gradients with counter signs have also been noticed due to the non-uniform thermal boundary condition of the wall.

Nusselt number (Nu)

The Nu implies the relation between the diffusive to

Table 1: Comprising results with current code and previous work.

Sl. No.	Parameters selected for code validation	Existing work	Present code	% in change
1	Ra = 10^5, Da = 10^{-1}, and Pr = 1 [30]	**4.385**	**4.276**	**2.40**
2	Pr = 10, Ra = 10^5, and uniform heating [31]	**8.610**	**8.580**	**0.34**
3	Ra = 10^5, Da = 10^{-3}, and φ = 0° [32]	**2.623**	**2.610**	**0.38**

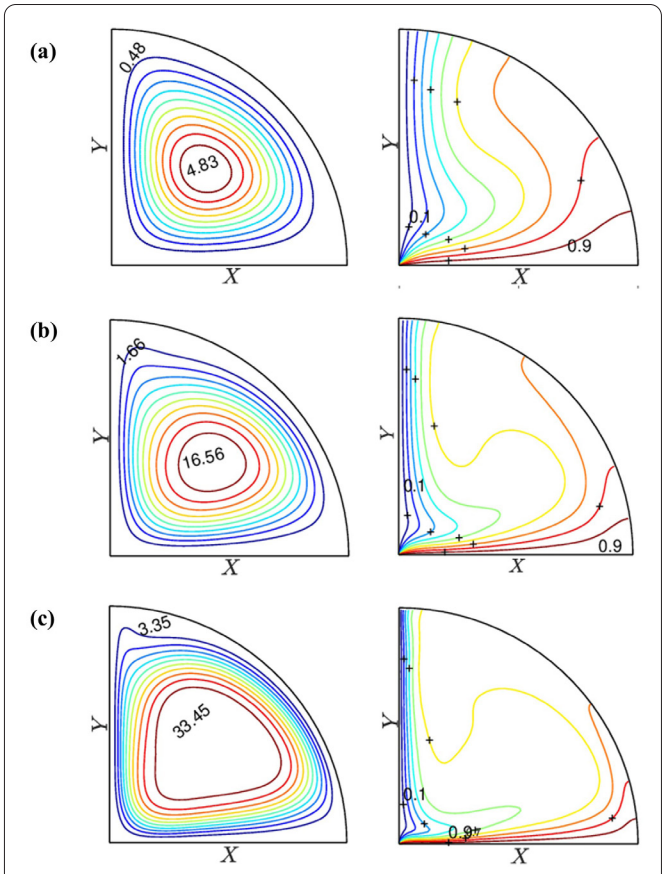

Figure 3: Vortex patterns (left) and non-dimensional temperature distribution (right) with Ra **(a)** Ra = 10^4, **(b)** Ra = 10^5, and **(c)** Ra = 10^6 at θ = 1.

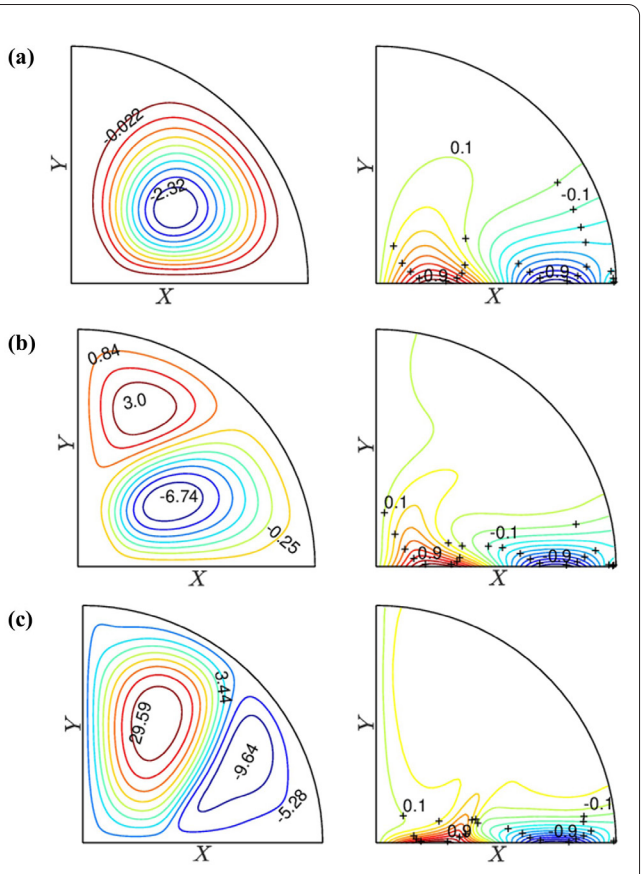

Figure 4: Vortex patterns (left) and non-dimensional temperature distribution (right) with Ra **(a)** Ra = 10^4, **(b)** Ra = 10^5, and **(c)** Ra = 10^6 θ = sin(2πX).

convective resistances of heat transfers, and is given by, $Nu = -\frac{\partial\theta}{\partial\eta}$ (η direction normal to the plane). The variations of local Nu along the bottom and left walls at different Ra under homogenous and uneven heating of the lowermost wall are exemplified in figure 5a and 5b; respectively. The figure shows a smaller value of local Nu at Ra = 10^4 due to lower convection strength. With a further escalation in Ra, convective method of heat transfer improves increasing buoyancy force that results in augmented local Nu. An elevated local heat transit rate is acquired at the lowermost left corner during uniform bottom wall heating as can be perceived from Nu plot in figure 5a. However, the distinction of local heat transfer rate is sinusoidal during uneven heating as usual for the sinusoidal nature of heating. For uniform heating, the local heat transference rate is obtained to be more than non-uniform heating.

As Ra is increased in case of uniform heating, the local Nu rises along the upright wall as the buoyancy force increases within the cavity that results in an increment of temperature gradient along the wall (figure 5b). The trend is the same for the uneven heating condition. However, the increment of heat transfer with the rise in Ra is to a greater extent foremost in the case of uniform heating. This may be due to the effects of the sinusoidal heating condition of non-uniform heating.

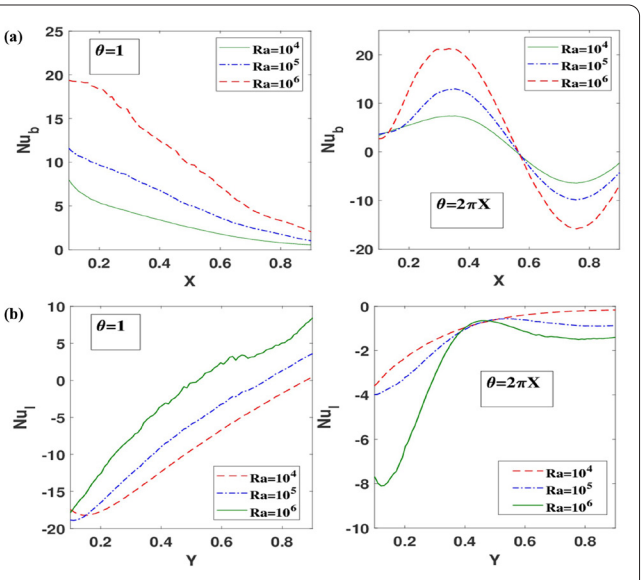

Figure 5: Local Nu along **(a)** bottommost wall and **(b)** upright wall with Ra.

Conclusion

This work investigates thermal flow characteristics within an air-filled quadratic enclosure heated from the below uniformly as well as non-uniformly, separately. Major inferences of the present investigation are listed below.

- Strength of stream function amplifies with Ra as buoyancy force rises inside the cavity.
- Increment in convective heat transfer within the cavity leads to an increase in thermal penetration as Ra is increased.

- Uniform heating conditions lead to a greater circulation strength than non-uniform heating.
- The strength of the circulation roll enhances 2 times for uniform heating, and 4.4 times for sinusoidal heating when Ra is varied from 10^5 to 10^6.
- Nu at the bottom wall along the wall under uniformly heating reaches its maximum at the left corner.
- Because of non-uniform bottom wall heating, the local Nu is sinusoidal in nature.

Acknowledgements

None.

Conflict of Interest

Authors have no conflict of interests to disclose.

Credit Author Statement

Saddam Hossain Mullick: Investigation, Writing - original draft preparation; Sourav Majumdar: Investigation, Writing - original draft preparation; Debabrata DasGupta: Writing - original draft preparation, Supervision; Pranab Kumar Kundu: Writing - review and editing, Supervision. All the authors read and approved the manuscript.

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