Analysis of Magneto-rheological Fluid Damper and Linearization of Semi-active Quarter Car Model

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Abstract
A vehicle with better suspension always provides extra satisfaction to the passengers. Active and semi-active suspension systems are meant to overcome the narrow comfort of passive suspension. Though active systems are superior in terms of performance, their cost makes it be used only in limited applications. Semi-active systems are the best compromise between active and passive systems. One procedure to achieve ‘semi-activeness’ is the use of magneto-rheological (MR) fluid in the system where the fluid property can be varied with a change in the magnetic field applied. The use of MR fluid in the damper for vehicular applications is presented in this study. The rheological characteristic of MR fluid prepared in-house is analyzed and the MR damper is characterized to understand the dynamic behavior of synthesized MR fluid. Then, the MR damper is represented mathematically using the modified algebraic model and is used in the quarter car model. Two road profiles are chosen for the analysis. Also, this study has attempted to address complexity arriving in the analysis of MR damper due to nonlinear hysteretic force characteristic using linearization toolbox in MATLAB Simulink.

Keywords
Magneto-rheological fluid damper, Quarter car, Modified algebraic model

Introduction
The MR fluid is a smart fluid that is widely used in damping devices. MR fluid consists of mainly magnetizable particles, carrier/base fluid, and additives as its constituents [1]. Magnetic particles are mixed in carrier fluid to form the base structure of fluids. When fluid is subjected to a magnetic field, magnetic particles align themselves to change the apparent viscosity. It causes the reversible transformation of fluid from liquid to semisolid form. This change of viscosity happens within a fraction of seconds. Few applications of MR fluid are in clutches, brakes [1, 2], dampers [2], beams [3], and mounts [4]. Various additives are added to overcome specific shortcomings related to MR fluid [5]. The viscosity of MR fluids can be modified by altering carrier oil properties, particle type, particle volume [2], magnetic field strength, and shear rate [5]. Preparation and variation in rheology of MR fluid with change in constituents was described in several research studies [6, 7]. MR fluid is widely used in the semiactive automobile suspension system. The suspension system in automobiles can be of three types, passive, semiactive, and active. Semiactive dampers give advantages of both passive and active dampers. It provides comparable performance like active suspension at a much lesser cost.

Modeling of MR damper refers to mathematical modeling of MR damper to capture its hysteretic dynamic behavior [8]. A comparative study between modified algebraic model (mAlg) and other parameteric models was reported...
in previous studies [9, 10]. The quarter car model [11–13] is the most widely used suspension model to evaluate suspension performance. The hysteretic nature of force characteristics of MR damper introduces nonlinearity in the analysis of MR fluid-based applications. Nonlinearity makes the analysis of such systems complicated. But currently, very little literature is available to address this challenge.

This study focuses on the preparation of MR fluid and its rheological testing. The study demonstrates the dynamic characterization of MR damper and the use of the less-explored mAlg to model the hysteretic force characteristics of MR damper. An attempt has been made for the linearization of a nonlinear semiactive quarter car suspension model using MATLAB Simulink. Response of linearized quarter car model is obtained for two road profiles and its accuracy is compared with the original nonlinear model.

**Experimentation**

**MR fluid preparation and damper development**

MR damper piston is manufactured using steel grade of AISI 1018 owing to its good magnetic permeability. The damper’s electromagnetic piston is wound using 300 turns of an AWG 24 diameter copper coil. The MR fluid was prepared in-house by using a definite amount of constituents. In the present work, CIP i.e., carbonyl iron powder was used as ferrous particles whereas silicone oil was the base oil. Along with these, bentonite clay was used as the additive. Initially, the base oil and the additive were mixed and stirred for about 4 hours for uniform distribution. The ferrous particles were then added to this mixture and stirred for about 8 hours. In this work, a 75% weight fraction of CIP was added to the mixture of 21% base oil and 4% of the additive. The MR fluid thus prepared has undergone characterization testing using the rheometer. The rheometer setup (Make: Anton Paar Ltd.) is shown in figure 1. The MR fluid thus prepared has undergone characterization testing using the rheometer. The MRD cell enables the fluid characterization study under the influence of the different magnetic fields. The fluid characterization was performed under constant shear rate (300 1/s) and by varying the current supply to the fluid from 0.01 A to 5 A (thus changing magnetic field). The shear stress and viscosity variation of the fluid under the influence of varying currents are shown in figure 2a and figure 2b, respectively. A considerable change in the MR fluid property is observed when the current supply was varied. Maximum shear stress of 37613 Pa was seen at 5 A current input and similarly at the same current, the viscosity of MR fluid was increased to 125450 mPas. Hence, the response of MR fluid towards the current variation was significant.

**Experimental setup and characterization of MR damper**

The damper testing setup is used to test the MR damper to obtain force characteristics. The experiment is conducted using the damper testing equipment (make: HEICO Ltd.) of 10 kN hydraulic capacity. This machine uses the potentiometer to measure the displacement, whereas load cell to measure the force. A data acquisition system is implemented to collect the measured data, and a controller is employed to change the input conditions. For testing, a sine wave with maximum amplitude of ±10 mm and a frequency of 2 Hz is employed. The current input to the damper is varied as 0.25 A, 0.5 A, 0.75 A, 1 A and off-state during testing.

Experimental data obtained from the above testing can be visualized with help of figure 3 of the force-displacement plot (Figure 3a) and force-velocity plot (Figure 3b). The force variation with displacement reveals the MR damper’s hysteretic behavior. Also, it shows that the peak force value goes on increasing with current input.

**Modeling of damper using mAlg**

To analyze the performance of the MR damper, various mathematical models have been used to simulate the MR
damper’s force characteristics. These models can be non-parametric or parametric. Parametric models are combination of mechanical components like spring, damper, etc., Bouc-Wen model, Dahl model, etc. are examples of parametric models. To overcome limitation of the basic models, new and modified models were proposed [9]. Another important type of model is the algebraic model. These models have simple mathematical expressions and have good modelling accuracy [9]. Current study uses a mAlg model. Less number of shape deciding parameters reduces the computational time during modeling process and the final response of the system adapting this algorithm be faster. Hence, can be effectively utilised in the control process.

The mathematical representation of the mAlg is,

\[ F(t) = f_0 + C_i \dot{x}(t) + f_r (2/\pi) \arctan \left( k \left( \dot{x}(t) - \dot{x}_r \text{sgn}(\dot{x}(t)) \right) \right) + m \ddot{x}(t) \]  

(1)

Where, \( F(t) \) is the force of MR damper; \( f_0 \) stands for preload, \( C_i \) indicates viscous damping, \( f_r \) is yield force, \( k \) is coefficient of shape, \( \dot{x}_r \) is hysteresis velocity, and \( m \) is virtual mass [8]. \( f_0 \) is responsible for moving entire curve up and down, \( f_r \) controls dynamic force range, \( C_i \) accounts for the slope of the whole curve, \( k \) controls slope, and \( \dot{x}_r \) is meant for controlling span of hysteresis loop at low velocity. \( m \) is used to control stretch of hysteresis loop corresponding to high velocity. The parameters of this model are found using genetic algorithm. Values of parameters for all the current values are listed in Table 1.

Optimization is employed to minimize the RMSE i.e., root mean square error in the experiment force value and the predicted value of force. By setting the RMSE as the objective function of genetic algorithm, the parameters of mAlg are found out [7]. Five sets of values of parameters corresponding to 0.25 A, 0.5 A, 0.75 A, 1 A and off state are generated using a genetic algorithm. An illustrative example of a comparison between experiment and theoretical force value for 0.25 A is shown in Figure 4.

**Quarter car suspension 2-DOF model**

The quarter car suspension model [11-13] is a 2-DOF model. It is used to understand how the suspension system will behave when subjected to different road profiles. It has two degrees of freedom, one at sprung mass while the other at unsprung mass. The conventional passive suspension has a constant force value due to dampers, but in semi-active suspension variable damping force is obtained (Figure 5).

The equation of motion for 2-DOF with semi active damper is given as:

\[ M_s \ddot{x}_s + F_d + K_s (x_s - x_u) = 0 \]  

(2)

\[ M_u \ddot{x}_u - F_d - K_s (x_s - x_u) + C_i (\dot{x}_u - \dot{x}_s) + K_s (x_u - x_s) = 0 \]  

(3)

In the above equation, \( M_s \) stands for sprung mass and \( M_u \) for un-sprung mass, \( K_s \) represents spring stiffness, and \( x_s, x_u, x_s \) stand for sprung mass displacement, un-sprung mass displacement and road profile displacement, respectively. \( F_d \) is force due to MR damper given by equation 1. Quarter car parameters given in Table 2 are used in this study [11].

**Linearization of quarter car using Simulink**

Vehicle suspension consists of spring and damping elements. When the passive damper is replaced by an MR damper, which is hysteretic, causes the whole model to become nonlinear. Hence analysis of such a system becomes very complicated. It needs the use of further linearizing techniques to convert the nonlinear system into an approximate linear system. For this study, two profiles of road, random road profile and sinusoidal road profile are considered.
Linearization of nonlinear system is based on concept of trim point or operating point. In this, operating point are value of input parameters for which system shows desired or predictable behaviour. System is then assumed to be behaving as linear system for input values close to operating points. MATLAB Simulink tool, Linear analysis tool are used to find these trimming points.

Further, this linearized model is used to find response of linearized system. On other hand, response of nonlinear system is obtained using MATLAB Simulink. First, Simulink model of quarter car is built using governing equations 1, 2, and 3. Then, response of this quarter car model is obtained for road profiles discussed below.

Random road profile (Figure 6a) is considered to be response of the 1st order linear filter to white noise [13] which is represented by

$$h(t) + a(t) = w(t)$$

(4)

Where, $w(t)$ is the white noise process and it is obtained by the following series.

$$w(t) = \sqrt{2} \sum_{k=1}^{N} \left( S_k \left( w_k \right) \Delta w \right)^{1/2} \cos \left( w_k t + \phi_k \right)$$

(5)

Where, $S_k(w) = \frac{\sigma^2 w^3}{\pi}$ is the one-sided PSD of road frequency $\omega_k$ with $w_k = w_0 + \left( k - 1/2 \right) \Delta \omega$ and $k = 1, 2, 3...N$, $\Delta w = \frac{w_k - w_0}{N}$, where $\omega_k$ is upper cutoff frequency and $\omega_0$ is lower cutoff frequency. $N$ represents the number of intervals. Also, $\sigma^2 = 3 \times 10^4$, $\alpha = 0.45$ and $V' = 16.67 \text{ m/s}$ and $w_k' = w_k + \delta w$ [13] in which $\delta w$ is small random frequency. It is distributed uniformly in $\Delta w' \ll \Delta w$ and $\phi_k$ with $\Delta w' \ll \Delta w$ and $\phi_k$ is the independent random phase.

It is distributed uniformly in 0 and $2\pi$. The parameters are $N=1000$, $w_0 = 0$ and $\omega_k = 2\pi \times 100$ and $\Delta w' = 0.05 \Delta \omega$ [13].

Sinusoidal profile was with a peak amplitude of 0.1 m and duration of 10 s (Figure 6b). Linearization of the nonlinear system is a mathematically challenging task. But MATLAB provides a toolbox that helps with linearization. In this study, the “Linear Analysis toolbox” is used for obtaining a linear system in Simulink. As an output of this linearization, the whole system is obtained in state-space form as given below.

$$\dot{x} = Ax + Bu$$

(6)

$$y = Cx + Du$$

(7)

Where, $x$ is the state vector, $y = [x_1, x_2, x_3, x_4]^T$. $y$ is output vector. $u$ denotes input vector. As an example, state space matrices for 0.75 A current case are as follows:

$$A = \begin{bmatrix} 0.0010 & 0.0011 & 0 & 0 \\ -0.0558 & -0.0534 & 0.0558 & 0.0534 \\ 0.0008 & 0 & 0.0011 & 0.0010 \\ 0.2743 & 0.2623 & -3.4946 & -0.4500 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 3.2203 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \text{and} \quad D = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The linearized quarter car is compared with the nonlinear model by using the road profile described in figure 6 and the results obtained can be visualized as shown in figure 7 and figure 8. A gap between non-linear and linearized responses is observed nearer to 2 second in figure 7. It was observed that the gap showed a narrowing tendency with increase in damping, i.e., with increase in current. Figure 8 shows the comparison between response of nonlinear and linearized quarter model to random road profile (Figure 6a).
It can be observed that the nature of response for non-linear and the linearized model almost traces a similar path but the differences can be observed in the peak values (both crest or trough). Table 3 gives maximum error in sprung mass displacement between nonlinear and linear quarter car model subjected to sinusoidal road profile and random road profile. Converting non-linear system into linear system provides an advantage that the stochastic optimal control theory and the standard random vibration theories can be directly applied [13].

Table 3: Max error in sprung mass displacement between linearized and nonlinear quarter car model.

<table>
<thead>
<tr>
<th>Current (A)</th>
<th>Max error in spring mass displacement (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Off</td>
<td>0.0482</td>
</tr>
<tr>
<td>0.25</td>
<td>0.0538</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0609</td>
</tr>
<tr>
<td>0.75</td>
<td>0.0399</td>
</tr>
<tr>
<td>1</td>
<td>0.0277</td>
</tr>
<tr>
<td>0.75</td>
<td>0.0025 (for random road profile)</td>
</tr>
</tbody>
</table>

Conclusion

The study presents the in-house preparation of MR fluid and dynamic analysis of MR damper. Initially, MR fluid of definite composition was prepared and the rheological characteristics of MR fluid showed the significant response of MR fluid when exposed to magnetic field. Characterization of MR damper with in-house prepared fluid showed at least twice the damping force at maximum current when compared to off-state. The study also showed the effective implementation of mAlg model to represent the hysteresis behaviour with minimum error. The complexity arising due to nonlinear damping force was addressed by the linearization of the nonlinear quarter car model. The comparison between the response of the linearized and nonlinear system was done by comparing the sprung mass displacement in both cases. The least error of 0.0025 m was recorded for random road profile at 0.75 A current. The study highlighted preparation, testing, and use of MR fluid in the damper for vibration control and the use of linearization technique to simplify the analysis of a nonlinear complex system.

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Conflict of Interest

Authors declare that there is no conflict of interest.

Credit Author Statement

N P Puneet: Experimentation; Swagat Kumbhar: Design, Data analysis; Hemantha Kumar: Writing - original draft preparation; K V Gangadharan: Writing - review and editing. All the authors read and approved the manuscript.

References

