

Fault Detection and Diagnosis of Rotor-ball Bearing System

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Abstract

Every production machinery with a rotating element for transmitting power utilizes bearings to minimize frictional losses while supporting the rotating element and preventing the element's motion in the direction of the applied load. This feature of the bearing makes it an essential element to watch for, as its failure may lead to low productivity of the production system and, if not taken proper care of, can even result in catastrophic failure of the production system. Consequently, in the present work, fault detection and diagnosis (FDD) of the rotor-ball bearing system is presented to diagnose the faults in the inner race, outer race, and balls of the bearing. As the vibration signals of the rotor-bearing system vary with healthy or different faulty conditions of ball bearing, these signals are used for FDD by using statistical feature extraction techniques to observe a recognizable pattern. These features' values are now further used to train Artificial Neural Networks (ANN) (Feed-forward Backpropagation Neural Network; FBNN and Cascade Feed-forward Backpropagation Neural Network; CFBNN) to enable an automatic fault detection and diagnosis system. The main drawback of the Neural Network is its computational time requirement which has been overcome by using statistical feature extraction techniques while utilizing the adaptive nature of the Neural Network. The results show that Cascade Feed-forward Backpropagation Neural Network gives better classification results and is more efficient than Feed-forward Backpropagation Neural Network.

Keywords

Fault detection and diagnosis, Rotor-ball bearing system, Statistical features, Feed-forward backpropagation neural network, Cascade feed-forward backpropagation neural network

Introduction

In the present scenario, every organization, in some way or another, depends upon the machine, and its failure leads to loss or loss of profit. Therefore, it's essential to accurately diagnose the fault and classify and rectify it. Every production machinery with a rotating element for transmitting power utilizes bearings to minimize frictional losses while supporting the rotating element and preventing the element's motion in the direction of the applied load. This feature of the bearing makes it an essential element to watch for, as its failure may lead to low productivity of the production system and, if not taken proper care of, can even result in catastrophic failure of the production system. Therefore, research in the fault diagnosis of bearing and its classification has been widely developed.

Various types of faults in the ball bearings can exist, like faults in the inner race, outer race, or balls of ball bearings. These faults may happen due to cracks, flaking, roughness, surface distress, corrosion, spalling failure, etc. For fault di-

agnosis of these faults, many fault diagnosis techniques have been applied in the literature. Yang et al. [1] used the intrinsic time-scale decomposition method for choice of features and the robust regression-variable predictive mode-based class discriminate method for pattern identification but have inherent limitations as these processes become more complex as the data increases to a very large value. In intrinsic time-scale decomposition method, as the original vibration signal is decomposed into several PRCs, it is also one of the problems if some of the external variables can severely affect PRC components, and thus, fault diagnosis will be more inaccurate. An artificial neural network with backpropagation for pattern recognition and classification into the faulty and healthy bearing with 93 percent accuracy was used by Ali et al. [2]. Sobie et al. [3] used the envelope method to extract features and convolutional neural networks to classify bearing into healthy and faulty conditions. The envelope method has the fundamental problem of over and under the envelope. Elasha et al. [4] collected acoustic data for bearing with minor outer race fault, major fault with inner race and outer race using an adaptive filter algorithm and the resultant signature was processed by using the envelope method, which has the problem of over and under the envelope. The authors used only the partial kurtosis method to differentiate between various bearing conditions. Maru et al. [5] collected vibrational data using an accelerometer for bearing contaminated oil and uncontaminated conditions. They used the RMS method only to distinguish between contaminated bearing operated conditions and uncontaminated bearing conditions. Halme [6] presented a paper focusing on wear debris analysis in oil-circulating lubrication systems. The research was focused on the change of color of the oil and its properties of light penetration, counting wear particle debris, and during the accelerated test, size distribution and changes of debris were also measured. This approach resulted in the classification of only two conditions, i.e., the healthy state and the faulty condition of bearing. Feng et al. [7] also used the amplitude envelope technique, which has the problem of over and under enveloping conditions.

From literature review, it is concluded that fault detection and diagnosis in ball bearings consider various fault detection techniques, including vibrational analysis, wear debris analysis, acoustic emission analysis, and temperature analysis. Out of these techniques, vibrational analysis is the most commonly used.

Feature extraction and selection of the most appropriate feature are vital. Researchers have used many statistical features such as Mean, Kurtosis, Standard Deviation, Peak-to-Peak Value, etc. Yet many statistical features such as Crest Factor, Impulse Factor, and Simple Sign Integral have not been utilized. Here, efforts are made to analyze the vibration signals of a rotor-bearing system having deep groove ball bearings based on the above statistical features. Further, these features are utilized as inputs to the FBNN and CFBNN to classify healthy and faulty conditions of the system.

Materials and Methods

Fault detection and diagnosis technique of rotor bearing system

Fault diagnosis is essential for maintaining a given

machine's satisfactory performance. A traditional fault diagnosis involves the following three steps:

1. Data acquisition,
2. Feature extraction, and
3. Fault identification and classification

Data acquisition

Vibrational data for bearing in healthy condition, bearing having fault in the inner race, fault in the outer race of the bearing, and bearing having ball fault have been taken from the Case Western Reserve University [8].

Feature extraction

A feature means a criterion through which fault can be identified and utilized further for the classification. Various statistical features are used in the present work for FDD of the rotor bearing system and are specified in table 1.

Classifications

After feature extraction, a specific pattern can be observed in the extracted features. The apparent, distinguishable features are then further used to train ANNs. The trained ANN is employed to classify the given data in four classes, i.e., fault in the inner race of the bearing, fault in the outer race of the bearing, ball fault of bearing, or healthy condition of the bearing. The ANNs used in this work are FBNN (Figure 1) and CFBNN (Figure 2).

Illustration

Data acquisition

The test setup available at Case Western Reserve University is shown in figure 3. The layout has a two-hp motor (left), a dynamometer (right), a torque transducer (centre), and controls (not shown). The motor shaft is supported by the 6205-2RS JEM SKF deep groove ball bearings in the test setup. The fault introduced are single points to the test bearings using EDM with fault diameters of 0.007 inches. An accelerometer was used to collect vibrational amplitude data by attaching it to the housing with a magnetic base. Data recorded were at the rate of 12,000 samples per second. Bearing specification and its defective frequencies are listed in table 2 and table 3. The vibration data signals obtained for

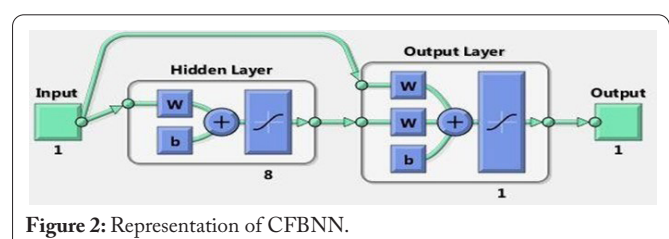
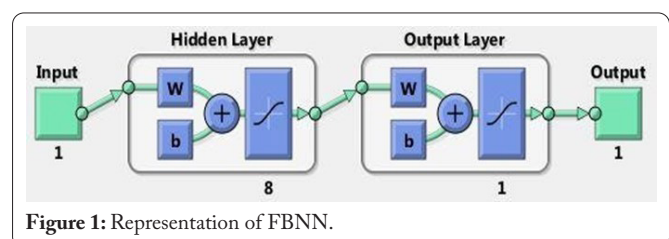


Table 1: Statistical features for FDD [9-12].

Statistical Feature	Description	Formula
Mean	the sum of values of sample data to the total number of samples	$\mu = \frac{\sum_{i=1}^N Y_i}{N}$
Mean Absolute Value	absolute average of the given sample	$MAV = \frac{\sum_{i=1}^n Y_i }{n}$
Standard Deviation	variation of a given quantity of sample from the Mean of a sample	$\sigma = \sqrt{\frac{\sum_{i=1}^N (Y_i - \mu)^2}{N - 1}}$
Variance	square of standard deviation and represents the absolute deviation of data	$\text{var} = \sigma^2$
Root Mean Square	amplitude modulated Gaussian random process	$Y_{RMS} = \sqrt{\frac{\sum_{i=1}^N Y_i^2}{N}}$
Kurtosis	fourth moment to find tailedness in the probability distribution curve	$\text{Kurtosis} = \frac{\sum_{i=1}^N (Y_i - \bar{Y})^4}{\sigma^4}$
Skewness	the third moment of distribution to measure the asymmetry of the probability distribution about its mean	$\text{Skewness} = \frac{\sum_{i=1}^N (Y_i - \bar{Y})^3}{\sigma^3}$
Crest Factor	the ratio of the maximum value of amplitude to the square of the mean of the root of absolute values	$CF = \frac{Y_{\max}}{Y_{rms}}$
Simple Sign Integral	the energy of the given segment	$SSI = \sum_{i=1}^n Y_i ^2$
Shape Factor	the ratio of RMS value to mean absolute value	$SF = \frac{Y_{rms}}{MAV}$
Impulse Factor	the ratio of the maximum amplitude of vibration to mean absolute value	$IF = \frac{Y_{MAX}}{MAV}$

healthy bearing, bearing with ball fault, bearing with inner race fault, and bearing with outer race fault are shown in figure 4. It can be observed from figure 4 that the amplitude of vibration varies for different conditions of ball bearings. However, to enable automatic FDD, it is more appropriate to convert the information in these vibration signals to forms, which can be further utilized to apply artificial intelligence techniques, like ANNs. Therefore, the obtained vibration signals are further divided into twenty parts used to determine the various statistical features described in table 1.

Feature extraction

Based on the vibration signals, the various features for

different ball bearings conditions are determined and given in table 4. It can be observed that the statistical features show variation corresponding to different states of the ball bearings. These features' values are now further used to train ANNs (FBNN and CFBNN) to enable automatic fault detection and diagnosis using the information of different features.

Classification

Now, the FBNN (Figure 1) and CFBNN (Figure 2) are applied to the determined features (Table 4). The Target values used for classification by FBNN and CFBNN are: i. 1 for healthy bearing.

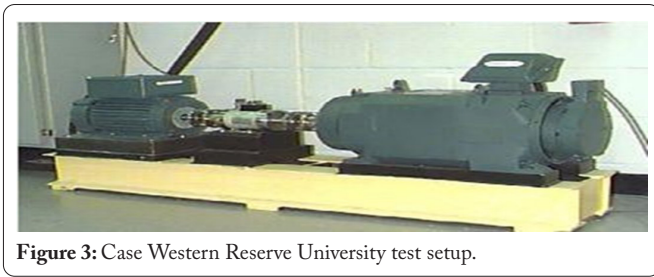


Figure 3: Case Western Reserve University test setup.

Table 2: Bearing specification bearing [8].

Inside Diameter	Outside Diameter	Thickness	Ball Diameter	Pitch Diameter
0.9843 inch (25 mm)	2.0472 inch (52 mm)	0.5906 inch (15 mm)	0.3126 inch (7.94 mm)	1.537 inch (39 mm)

Table 3: Defect frequencies (Hz) [8].

Inner Ring	Outer Ring	Cage Train	Rolling Element
5.4152	3.5848	0.39828	4.7135

- ii. 3 for fault in the inner race.
- iii. 5 for bearing having in fault in the outer race.
- iv. 7 for bearing having ball fault.

To classify the bearing condition for above target values, the ranges (upper and lower limits) are defined by considering a margin of 0.5 around the target class values. If the outputs from FBNN and CFBNN are in between the following ranges, the bearing condition is classified accordingly:

- i. 0.9 to 1.5 Healthy bearing.
- ii. 2.5 to 3.5 Inner Race fault.
- iii. 4.5 to 5.5 Outer Race fault.
- iv. 6.5 to 7.0 Ball fault.

If any of the above conditions are satisfied, the results can be predicted; else, the results cannot be predicted.

FBNN and CFBNN have been used with one input neuron,

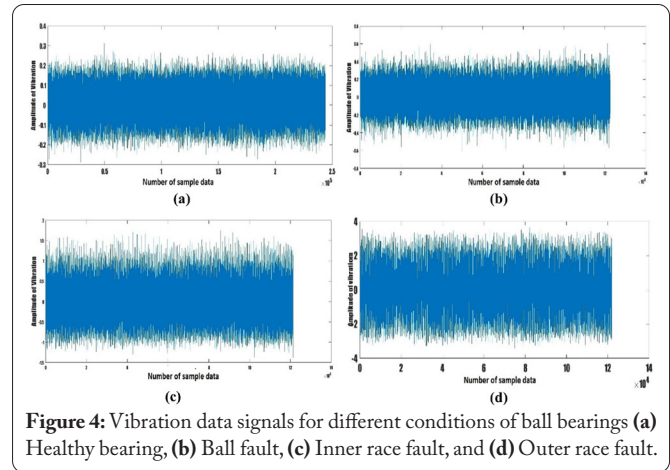


Figure 4: Vibration data signals for different conditions of ball bearings (a) Healthy bearing, (b) Ball fault, (c) Inner race fault, and (d) Outer race fault.

one output neuron, and two hidden layers of neurons. The gradient descent momentum training method is used here with the Simple Sign Integral function as a feature. Simple Sign Integral feature is selected for initial training among the various determined features, as the deviation in values of Simple Sign Integral for different bearing conditions is maximum.

The tang transfer function is used for its better pattern recognition performance. For weight adjustment, the adaptive learning function is used with gradient descent momentum as the adaptive learning function. To identify the optimum number of neurons for classification accuracy, the number of neurons in each hidden layer varies from two to ten for feed-forward backpropagation, and from eight to ten for cascade feed-forward backpropagation (Table 5), respectively. Based on the best classification accuracy, ten hidden neurons are selected for FBNN, and eight are selected for CFBNN. Here, the classification accuracy implies how well a network can classify bearing in healthy condition, bearing having a faulty inner race, bearing having a defective outer race, and bearing having faults in balls of bearing. If the network can classify all four cases, its accuracy is 100 percent; if it is unable to classify one case, its accuracy is 75 percent; if unable to classify two cases, its classification accuracy is 50 percent, and if unable to classify three its classifications accuracy is 25 percent, and if unable to classify all four cases, its classification accuracy is 0 percent.

Table 4: Statistical features range for various bearing conditions.

Statistical Features	Healthy Bearing	Inner Race Fault	Outer Race Fault	Ball Fault
Mean	0.01	0.02 - 0.03	0.012 - 0.015	0.01 - 0.015
Mean Absolute Value	0.06	0.4	0.2	0.1
Standard Deviation	0.07	0.67 - 0.69	0.29	0.13 - 0.14
Variance	0.005	4.2 - 4.8	0.08	0.02
Root Mean Square	0.07	0.63 - 0.69	0.28 - 0.29	0.13 - 0.14
Kurtosis	2.6 - 2.8	5 - 5.8	7.4 - 7.8	2.8 - 3
Skewness	0.21 - 0.3	3 - 3.6	1.4 - 1.7	0.46 - 0.5
Crest Factor	2.9 - 3.4	4.6 - 5.2	5 - 5.8	3 - 4
Simple Sign Integral	25 - 29	2112 - 2430	411 - 444	90 - 100
Shape factor	1.2	1.6 - 1.7	1.4 - 1.5	1.3 - 1.4
Impulse factor	3.5 - 5.2	1.6 - 1.7	7 - 8	4.4 - 5

Table 5: Comparisons of FBNN and CFBNN with neuron variations for FDD.

Number of Hidden		Training function	Feature	NetworkType	Transfer Function	Adaption Learning Function	Accuracy %
Neurons in each layer	Layer						
2	2	traingdm	SSI	FBNN	Tansig	learngdm	75
4	2	traingdm	SSI	FBNN	Tansig	learngdm	75
6	2	traingdm	SSI	FBNN	Tansig	learngdm	50
8	2	traingdm	SSI	FBNN	Tansig	learngdm	25
10	2	traingdm	SSI	FBNN	Tansig	learngdm	100
8	2	traingdm	SSI	CFBNN	Tansig	learngdm	100
10	2	traingdm	SSI	CFBNN	Tansig	learngdm	25

After selecting the optimum number of hidden layer neurons for FBNN and CFBNN, further analysis is proceeded based on the optimum network structures, and the performances of the networks are measured by using mean square error as the performance parameter.

From the graphs in figure 5, it can be seen that the validation performance of FBNN is not showing much improvement with the increase in the number of epochs. Further, CFBNN gives a better validation performance than FBNN for the same number of epochs, and its performance is also improving with the increase in the number of epochs.

The best-demonstrated validation performance of CFBNN is 6.37×10^{-13} at 411 epochs. Hence, CFBNN is the preferred ANN for the current FDD approach. From figure 6, it is found that for the optimum number of neurons along with other network parameters described above, the reliability of both the ANNs are approximately 100 percent. But as the performance of CFBNN is better than FBNN, CFBNN with the proposed architecture has been used in further analysis.

Results and Discussion

The CFBNN architecture is now further utilized for FDD based on all the statistical features. Table 6 shows the comparison of network performance based on some sample values of different statistical features. The sample features' values denote a particular bearing condition based on the previously determined ranges (Table 4). The corresponding outputs can be used to categorize the bearing conditions per the approach in section 3.3. The performances are summarised in table 5 to obtain a measure of the correct diagnosis capabilities of CFBNN based on a particular statistical feature.

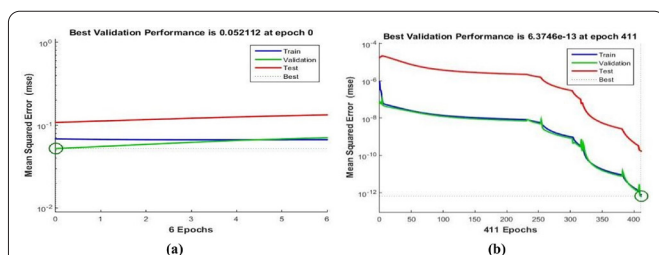


Figure 5: Performance of ANNs (a) FBNN and (b) CFBNN.

It can be seen that Simple Sign Integral and Mean Absolute Value give correct diagnosis classification results for all cases, whereas Crest Factor and Impulse Factor give wrong diagnosis results only for Inner race fault and Healthy bearing, respectively. Shape Factor and Variance only provide a correct diagnosis for inner race fault, and all other conditions are classified wrong. Mean Value gives false prediction results for outer race fault, ball fault, and correct diagnosis for healthy bearing and Inner race fault. Standard deviation gives accurate results for outer race fault and ball fault. The diagnosis results are wrong for healthy bearing and ball fault by using Kurtosis as a feature. Using the Root Mean Square value, Healthy Bearing and Inner race fault in the bearing are diagnosed wrong. Results show that Skewness should not be used as a

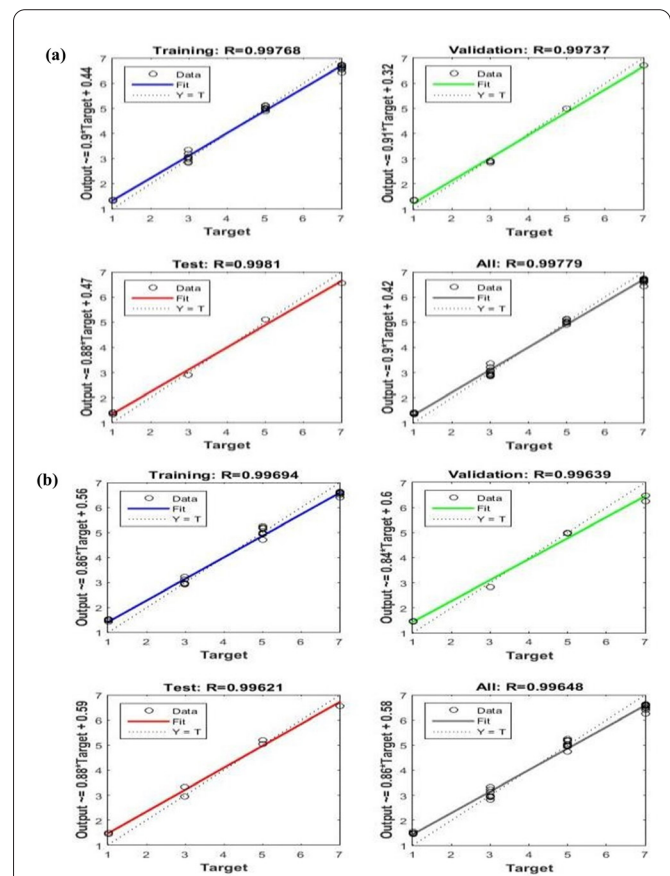


Figure 6: Reliability of ANNs (a) FBNN and (b) CFBNN.

Table 6: Comparison of various features.

Feature	Bearing Condition	Input	Output
Mean	Healthy	0.01,0.01,0.01	1.14,1.14,1.14
	Inner Race Fault	0.02,0.02,0.02	3,3,3
	Outer Race Fault	0.012,0.012,0.012	5,5,5
	Ball Fault	0.01,0.01,0.01	6.8,6.8,6.8
Mean Absolute Value	Healthy	0.06,0.06,0.06	1.1,1.1,1.1
	Inner Race Fault	0.4,0.4,0.4	3,3,3
	Outer Race Fault	0.2,0.2,0.2	5,5,5
	Ball Fault	0.1,0.1,0.1	6.8,6.8,6.8
Standard Deviation	Healthy	0.07,0.07,0.07	1.9,1.9,1.9
	Inner Race Fault	6.9,6.8,6.9	3.9,3.9,3.9
	Outer Race Fault	2.9,2.9,2.9	5.3,5.3,5.3
	Ball Fault	1.4,1.4,1.4	6.12,6.12,6.12
Variance	Healthy	0.05,0.05,0.05	3.9,3.9,3.9
	Inner Race Fault	4.6,4.3,4.3	3,3,1,3.1
	Outer Race Fault	0.08,0.08,0.08	2.8,2.8,2.8
	Ball Fault	0.02,0.02,0.02	5,5,5
Root Mean Square	Healthy	0.07,0.07,0.07	1.9,1.9,1.9
	Inner Race Fault	6.9,6.8,6.9	3.9,3.9,3.9
	Outer Race Fault	2.9,2.9,2.9	5.3,5.3,5.3
	Ball Fault	1.4,1.4,1.4	6.12,6.12,6.12
Kurtosis	Healthy	2.62,2.7,2.66	1.9,2.3,2.6
	Inner Race Fault	5.4,5.8,5.4	2.9,3,2.9
	Outer Race Fault	7.6,7.4,7.5	5,5,5
	Ball Fault	2.9,2.8,2.9	2.8,4.6,6.3
Skewness	Healthy	-0.02,0.02,-0.05	4.2,4.6,3.5
	Inner Race Fault	0.04,0.01,0.03	2.5,2.5,2.7
	Outer Race Fault	0.2,0.2,0.2	2.7,2.5,2.5
	Ball Fault	-0.002,0.009, -0.014	5.5,5.5,5.5
Crest Factor	Healthy	3.15,3.5,3.5	2.7,3.2,3.2
	Inner Race Fault	5,5,5	3.2,3.2,3.2
	Outer Race Fault	5.8,5,5	5.7,3.2,3.2
	Ball Fault	3.8,3.8,3.8	6.1,6.1,6.1
Simple Sign Integral	Healthy	27,27,28	1.5,1.5,1.5
	Inner Race Fault	2397,2342,2181	3.1,3,2.9
	Outer Race Fault	429,423,426	5,4.9,4.9
	Ball Fault	99,98,98	6.5,6.5,6.5
Shape Factor	Healthy	1.2,1.2,1.2	3.9,3.9,3.9
	Inner Race Fault	1.7,1.7,1.7	3,3,1,3.1
	Outer Race Fault	1.4,1.4,1.4	2.8,2.8,2.8
	Ball Fault	1.4,1.4,1.4	5,5,5
Impulse Factor	Healthy	3.8,4.3,4.4	1.1,4.2,5.5
	Inner Race Fault	1.7,1.7,1.6	3,3,2.97
	Outer Race Fault	8,8,7	4.99,4.99,5
	Ball Fault	5,5,5	6.4,6.4,6.4

feature as it can lead to the wrong classification in all cases. These findings are summarized in table 7. It is also observed that Ball Fault is the most difficult fault to identify. Further, the different features' values for ball fault are near to the values for healthy bearing. Therefore, it can be concluded that the ball fault has the least detrimental effect on the rotor-bearing system performance.

Conclusion

The current work presents rotor-bearing FDD using vibration amplitude data. The proposed FDD algorithm involves ANNs utilizing the statistical features of the vibration data. Various statistical features such as Kurtosis, Skewness, Root Mean Square Value, Simple Sign Integral, etc., are used to extract information for four ball-bearing conditions. Among the various features calculated, the deviation in Simple Sign Integral is maximum; therefore, it is used as an initial feature for neural network parameter decision. With this initial feature, the FBNN and CFBNN are used to identify different bearing conditions, and CFBNN provides better performance. Therefore, CFBNN is further utilized for FDD with all the features. Results show the excellent performance of the proposed FDD algorithm. The best features for FDD are Simple Sign Integral and Mean Absolute Value, as the different bearing conditions are diagnosed correctly in all the cases. It is also observed that Ball Fault is the most difficult fault to identify, and it has the least detrimental effect on the rotor-bearing system performance.

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None.

Conflict of Interest

The authors declare that they have no conflict of interest.

Credit Author Statement

Prince Shukla: Conceptualization, Methodology, Resources, Investigation, Formal analysis, Writing - original draft preparation, Writing - review and editing; Praveen Kumar Agarwal: Conceptualization, Methodology, Writing - review and editing, Supervision. All the authors read and approved the manuscript.

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Table 7: Comparison of correct diagnosis capabilities of statistical features.

Feature	Wrong Diagnosis For
Mean	Outer Race Fault, Ball Fault
Mean Absolute Value	None
Standard deviation	Healthy Bearing, Inner race fault
Variance	Healthy, Outer Race Fault, Ball Fault
Root Mean Square alue	Healthy, Inner race fault
Kurtosis	Healthy, Ball Fault
Skewness	Healthy, Inner Race Fault, Outer Race Fault, Ball fault
Crest Factor	Inner Race Fault
Simple Sign Integral	None
Shape Factor	Healthy, Outer Race Fault, Ball Fault
Impulse Factor	Healthy Bearing

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