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Research Article

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Analysis of the von Mises Stress for Edge and Embedded Cracks in Statically Loaded Planar Elements

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Abstract

The present study is based on the determination and analysis of stress concentrations due to cracks in planar elements under static loading. It is pragmatic that finite element analysis (FEA) can efficiently determine the stress at a point near the crack tip. At present computer aided engineering (CAE) tools are at hand which can efficiently perform the FEA. In the present work the FEA is performed in ABAQUS platform. For the validation of the present work a Finite element model of a plate with two semicircular holes on opposite edges is developed by using the dimensions and other conditions as stated in a refereed publication. The present result is juxtaposed with the earlier published result in refereed publication. After that a planar model with crack is developed in ABAQUS and the model is simulated for different crack locations under static loading. The plate is fixed under different boundary conditions. The results are plotted against the crack locations to observe the effect of edge cracks and embedded cracks at different locations. It is noted that in every situation a regular pattern in between the von Mises stress value and crack location may not be obtained. Hence FEA is very essential for checking the stress in designing an element when any kind of discontinuity is present in the element.

Keywords

Finite element analysis, Crack, Computer aided engineering, von Mises stress, Stress concentration

Introduction

It is evident that the crack decreases the capability of a structural element to withstand the applied load. Hence it is utmost important to estimate the effect of cracks in structural elements. The stress concentration factor (SCF), K_t, is a fundamental parameter for anticipating the decrease in the load supporting capacity of a structural element due to crack. The values of 'K_t' are usually found in engineering reference manuals to compute the concentration in stress near the geometric discontinuities like an open crack. Then the estimated stress value is used to calculate the maximum stress. By knowing the value of maximum stress appropriate strengthening or repairing procedures can be recommended to avoid catastrophic failure. Here few of the relevant literatures are reviewed where the researchers have worked regarding the determination of stress and strain in the vicinity of any kind of geometric discontinuity. Brooks et al. determined the effect of a drill hole on long bone's durability and on the fracture resulting from applied torsion loads [1]. Kuang et al. studied the formation of fatigue cracks due to stress concentrations in tube-shaped joints [2]. Molski and Glinka, presented that theoretical "K_t" (SCF) can be determined by the unit elastic strain energy at the root of the notch [3]. Crews et al. performed a 2D finite-element study of stress in a laminate that is loaded using a steel pin [4]. Nedele and Wislom analyzed that the stress concentration in composite plates depends on hypothetical fiber bonding conditions [5]. Troyani et al. showed that the theoretical SCF is reliant on the length of the element in addition to other standard geometric parameters [6]. Noda and Takase derived an exact solution to determine the SCF for fillet in a bar [7]. Toubal et al. used an electronic speckle pattern interferometer to determine the stress concentration in terms of strain field for disc-shaped hole in a composites plate under tension [8]. Pilkey and Pilkey presented numerous results of "K_t" for different kind of geometries [9]. Yang et al. shown that the stress concentration factors at the root of a notch in a plate surface are functions of thickness [10]. Kubair and Chandar numerically investigated the effect of the material property on the SCF for a disc-shaped hole [11]. Rezaeepazhand and Jafari showed that the "K_t" can be altered in pierced plates by introducing some cutouts [12]. Mohammadi et al. measured the stress concentration factor around a disc-shaped hole in an infinite plate under uniform biaxial tension and pure shear [13]. Darwish et al. introduced a particular equation for the SCF calculation of a uniaxially loaded isotropic plate [14]. Sburlati presented a logical solution to reduce the SCF around a disc-shaped hole in an isotropic uniform plate under far-field uniaxial loading [15]. Hamanda et al. examined and analyzed the joint between the rib and plate in a T-shaped rigid body [16]. Yathisha et al. represented that the stress concentration factor for elliptical holes in plates made of fibers oriented at 45° is minimum [17]. Nguyen and Becker determined the stress concentration factors for bolt joints in orthotropic plates [18]. The literature review discusses about the research works which have been carried regarding determination of stress concentration and it also shows that the concentrated stress can cause premature failure of an element. The present research is an addition to the above discussed field. The research work aims to measure the increase of von Mises stress for edge cracks and embedded cracks in planar element under static loading.

Methodology

In this present research work, FEA is performed by using a CAE tool. It is well known that FEA can give approximate solutions of complicated differential equations which cannot be solved by direct methods. The methodology of the present FEA can be elaborated by splitting the whole method into three categories namely Pre-processing, Processing, and Post-processing.

Pre-processing

At first 3D planar deformable parts are created in the part module. For the validation the geometry is sketched according to the required shape to create the semicircular hole at the two opposite edges in order to get same plate element which is used for analysis in the literature [9]. The considered material properties are loaded in material library. The material properties are assigned on the created part by creating section in property module. The independent instances are created in the assembly module. To create the crack at first partitions are made at the locations where the crack will be assigned. Then in the interaction module the seam crack is assigned on the partitions. The part is discretized by quad-dominated elements with approximate global size 0.0015.

Processing

A static general step is created and the necessary boundary conditions e.g., to make one end fixed the 'Encastre' boundary condition is applied at one end of the planar element. Then the load is applied either at one node as concentrated static load or along the top or bottom edge as equally distributed pressure to apply the uniformly distributed load. Then the job is created and submitted.

Post-processing

After the simulation the result module is opened to see the deformed view. The required deformed shapes are obtained. Then by using 'query' the required results are taken at the specific nodes. Among the different theories of failures, the von Misses stress theory is an efficient theory to predict yielding in material under loading. It is calculated based on the deviatoric strain energy of the deformed body. The strain energy in the body in an element of unit volume can be articulated as in equation (1).

Ûdå ∫ (1)

For those materials which pose linear elastic property the equation (1) can be articulated as equation (2a).

ûå= (/) (2a)

In tensor form the equation can be written as

Û:å= $\left(\frac{1}{2}\right)$ (2b)

$\sigma = \sigma_h + \sigma', \in = \in_h + \in'$, by substituting this in to equation (2b).

$$U = \left(\frac{1}{2}\right)\sigma:\in = (1/2)(\sigma_h + \sigma'):(\in_h + \in') = (1/2)(\sigma_h:\in_h + \sigma_h:\in' + \sigma':\in_h + \sigma':\in') \quad (3a)$$

The double dot product between $\sigma_h, \in'$ and $\sigma', \in_h$ are zero. So, the equation (3a) reduces to

$$U = (1/2)(\sigma_h:\in_h + \sigma':\in') \quad (3b)$$

The von Mises stress is estimated by using the deviatoric part of the above equation which is $\left(\frac{1}{2}\right)\sigma':\in'$

From the linear hook's law,

$$\in' = \frac{1}{2G}\sigma'$$

So, the deviatoric strain energy $U' = \frac{1}{4G}\sigma':\sigma'$,

Now by considering a scalar stress value for the stress sensor the stress tensor can be written as $\sigma_r = \sqrt{\sigma':\sigma'}$

To calculate the von Mises stress, σ_r has to be determined for uniaxial tensile loading.

The state of stress at a point in uniaxial loading condition is:

$$\sigma = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and the hydrostatic stress } \sigma_h = \frac{\sigma}{3}.$$

Then the deviatoric stress

$$\sigma' = \begin{bmatrix} \sigma - \frac{\sigma}{3} & 0 & 0 \\ 0 & -\frac{\sigma}{3} & 0 \\ 0 & 0 & -\frac{\sigma}{3} \end{bmatrix},$$

So, $\sigma' : \sigma' = \left(2\sigma^2\right)/3$

$$\aleph_{\text{von Mises}} = \sqrt{\left(\frac{3}{2}\right) ' : '}$$

Results and Discussion

As it is stated in the introduction section that the purpose of the present work is to determine the rise in stress due to a crack in structural elements. In this work ABAQUS CAE is used to calculate the von Mises stress in different elements. First, the present theoretical work is validated by the published result in the book of Pilkey and Pilkey [9]. To validate the present theoretical work a plate element with two semicircular holes on two opposite sides is developed in the ABAQUS platform by using the dimensions as given in the refereed book [9]. The loading condition boundary conditions are kept the same as considered in the published literature. The maximum stress is observed near the semicircular discontinuity. The maximum stress is measured for different values and then the maximum stress values are divided by the nominal stress values to obtain the stress concentration factor. The comparison between present results with the published result in the book [9] is shown in figure 1.

Analysis of stress for crack

Here a planar element 100 mm long, 50 mm in breadth, made of aluminum (Material type isotropic, Young's modulus = 69 GPa, Density = 2789 kg/m^3, and Poisson's ration = 0.29 is considered. The edge cracks are considered perpendicular to the top edge of the plate such that the crack may open and close during loading depending on its location. Subsequently, a few embedded cracks are also analyzed. It is noted that the embedded crack in the considered element does not open when the load is not directly in mode I nature. The element is analyzed in both cantilever and both end fixed conditions. The element is loaded by a concentrated static load (500 N) at either middle or at the open end. In another run the element is loaded under UDL (5 N/mm). Figure 2a to 2d are the deformed shapes of the element for various loading, boundary conditions and crack locations. From figure 2a and 2b it is clearly observed that opening and closing of crack depends on the loading condition even under static condition. As in figure 2b, since the load is applied on the opposite side to the crack as a result crack remains closed. Therefore, it can be easily understood that the effect of crack is not same for both loading conditions. Figure 3 shows the variation of stress for different crack locations near the edge of the element for both the loading types. In figure 3 it is clearly observed that when the loading is on the same side to the crack then the characteristic of variation of stress with crack location is not predictable. But when the loading is on the opposite side then the relationship between variations of stress with crack location can be approx-

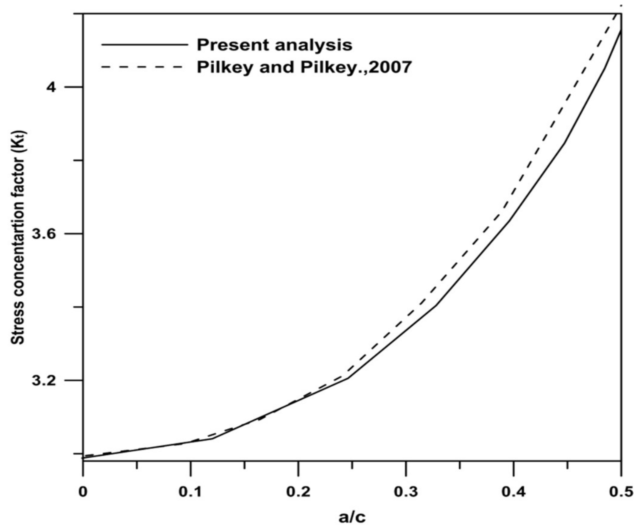

Figure 1: Comparison of present results with the published result in refereed article [9].

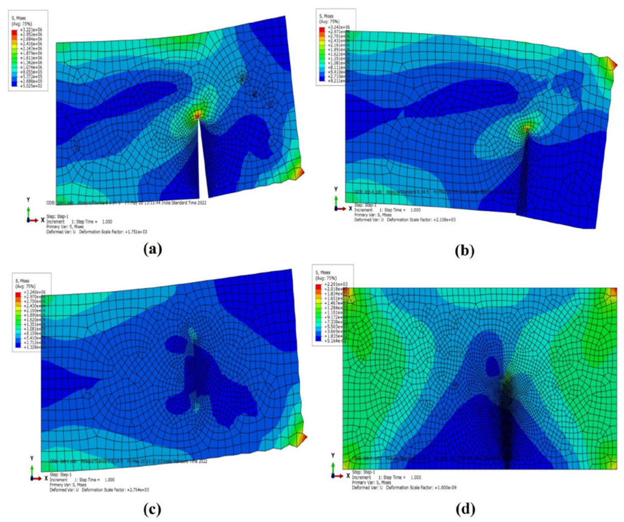

Figure 2: **(a)** One end fixed and concentrated load (500 N) is applied at Open end on the same side to the edge crack. **(b)** One end fixed and concentrated (500 N) load is applied at Open end on the opposite side to the edge crack. **(c)** One end fixed and concentrated load (500N) is applied at Open end, the crack is embedded. **(d)** Both ends fixed and UDL (5 N/mm) load is applied at top edge opposite to the edge crack.

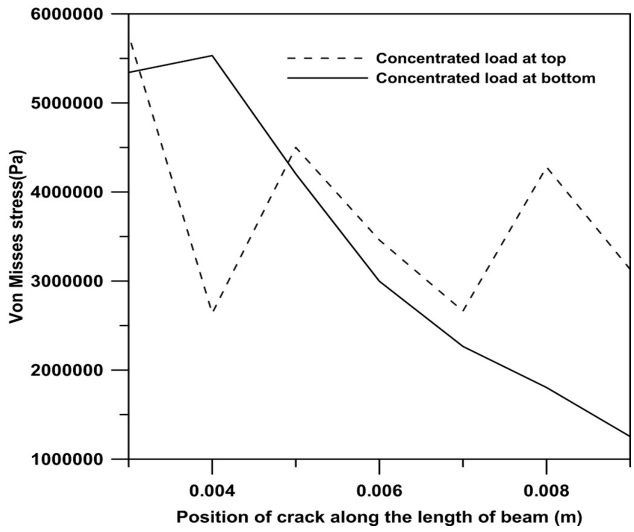

Figure 3: Variation of von Mises stress for the varying crack locations in one end fixed planar element under concentrated static load at open end.

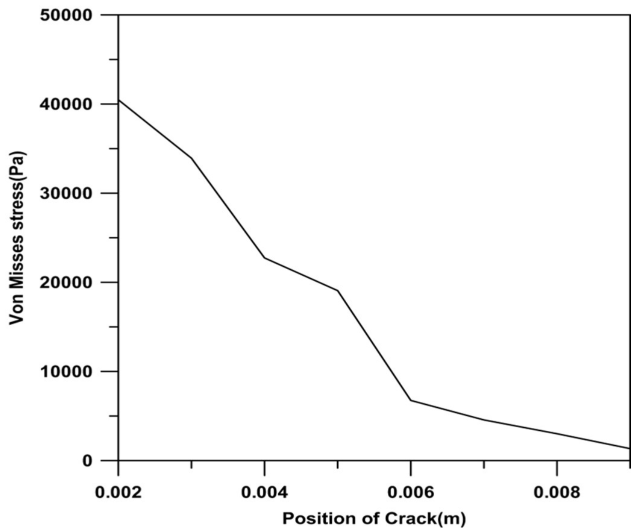

Figure 4: Embedded crack in one end fixed element under point load.

imated by a curve of higher order. As per the present observations during the study it is observed that when the loading is on the same side as the crack then the crack remains open. But the mouth size of the opening is different for different locations of crack as a result the generated stress is also different. Due to that the abrupt changes in the value of stress due to the location of cracks have come. But when the crack is on the opposite edge to the loading then the crack does not open as a result the generated stress follows a regular pattern with crack locations. Figure 2c represents the deformed shape of the finite element model of the element when the crack is embedded, and the load is applied at open end keeping opposite end fixed. The corresponding variation of stress with crack location is shown in figure 4. Where a relationship can be observed that when the crack is near the fixed end then the stress is more, but abrupt changes are also observed. Here it is observed during analysis that at some locations the embedded crack creates more space compared to the other locations as a result abrupt variation in the value of stress comes.

Figure 5 shows the effect of crack in raising the value of von Mises stress due to different crack locations under UDL on the opposite edge to the crack. It is observed that the effect of crack is more near the fixed end. In figure 6 the variation of von Mises stress against crack location for both end fixed element under UDL on the opposite edge to the crack is shown. Since the boundary condition at both ends are same due to that symmetric character with respect to the mid span is observed. Here it is noted that the effect of crack is minimum at the middle and maximum at the two ends.

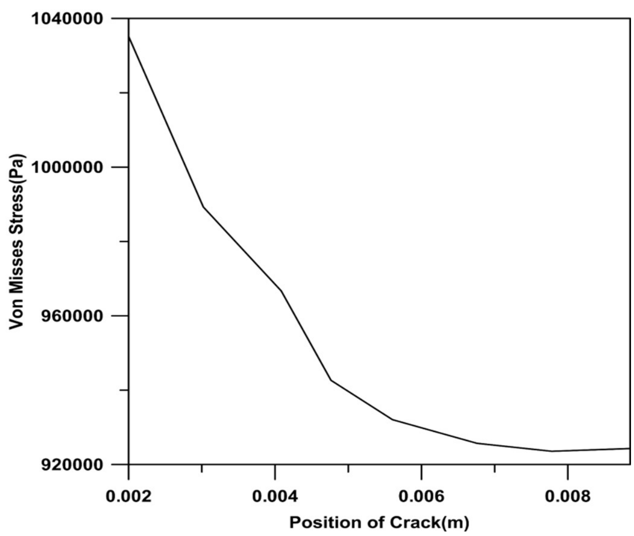

Figure 5: Edge crack in one end fixed element under UDL.

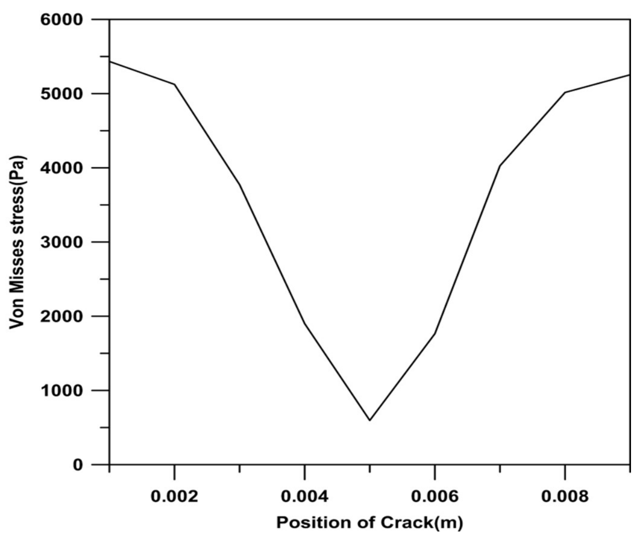

Figure 6: Edge crack in both end fixed element under UDL.

Conclusion

The increase in von Mises stress due to crack in statically loaded planar element under different boundary conditions are determined. The analysis is performed for both the concentrated load and uniformly distributed load. It is observed when the element is fixed at one end and the load is acting on the same side to an edge crack then the variation of von Mises stress with the location of crack is not following any regular pattern. It is discussed that since the opening of crack is different at different locations, the same crack is behaving like different geometric discontinuities at different locations. A regular pattern is observed for the other conditions such as when the loading is on the opposite edge to crack, when both end fixed element is under UDL to the opposite edge of the edge crack, etc. To authenticate the present FEA, a finite element model of a plate with two semicircular holes on opposite edges is prepared with the dimensions and conditions as presented in a published book and then the present result is compared with the published analytical result in the book. Indeed, this present work will aid in the section of solid mechanics. The present research work reveals that at every condition the stress concentration value cannot be predicted easily by usual theoretical calculations such as when an element with one end fixed is loaded in the same side to the crack. Therefore, in every designing work a finite element analysis is very essential to check the values of stress.

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None.

Conflict of Interest

No conflict of interest.

Credit Author Statement

Amartya Bose: Experimentation, Data analysis; Bikash Panja: Writing - original draft preparation; Sumit Chabri: Writing - review and editing; Akhtarujjaman Sarkar: Writing - original draft preparation; Ankesh Samanta: Writing - review and editing; Goutam Roy: Experimentation, Data analysis, Supervision. All the authors read and approved the manuscript.

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