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Nonlinear Transient Analysis of the Plate with Active Constrained 0-3 Viscoelastic Composite Layer Using Fractional Order Derivative Model

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Abstract

In this article, the nonlinear transient analysis of plate with active constrained 0-3 viscoelastic composite (VEC) layer is done. The ascendancy of the structural plate on hybrid damping control is analyzed. For the analysis, initially, closed-loop finite element (FE) modelling was done using von Kármán nonlinearity, layer wise first-order shear deformation theory (FSDT), and velocity feedback control strategy. The viscoelastic material (VEM) phase within 0-3 VEC is modelled using the fractional order derivative (FOD) method. To study the ascendancy of 0-3 VEC, initially, nonlinear transient responses of structural plate are determined at several geometric parameters of 0-3 VEC. The observations from the results acknowledge that the geometric parameters of 0-3 VEC affect the attenuation in vibration-amplitude. Thus, optimal geometric parameters of 0-3 VEC are estimated according to the utmost performance index. The results indicate that the active constrained 0-3 VEC may be advantageous compared to conventional VEM to control the geometrically nonlinear vibrations of structural plate.

Keywords

Viscoelastic composite, Nonlinear vibration control, Fractional order derivative, Finite element method, Hybrid damping control

Nomenclature

a is Length of structural plate, b is Width of structural plate, h is Thickness of substrate plate, h_c is Thickness of active piezoelectric constraining layer, Δ is Space between two consequent graphite blocks, h_v is Top/bottom thickness of VEM phase, n_g is Number of graphite blocks, h_d is Thickness of constrained layer, k_d is Velocity feedback control gain, I_d is Performance index.

Introduction

VEMs have viscous and elastic characteristics and magnificent energy absorption/dissipation capacity during transient load. So VEMs are commonly used in the suppression of structural vibration. However, active constrained layer damping (ACLD) is well known hybrid damping control of structural vibration [1]. It consists of VEM, constrained by substrate and active piezoelectric constraining layer. These structures are substantially utilized in various engineering applications like spacecraft, aircraft, trains, cars, and wind turbine blades [2]. Due to its enormous properties like high strength-to-weight ratio and vibration suppression capability, significant research has been done in this area [3-6]. Besides, VECs have also been utilized as damping material in structural applications [7-14].

However, these VEMs are usually modelled in the frequency domain and time domain. In this queue, various mathematical methods like Voigt model, fractional order derivative (FOD), anelastic displacement field, Golla-Hughes-McTavish, augmenting thermodynamic fields, Maxwell method [14-18] have been introduced for the modelling of viscoelastic damping materials. Out of these models, the FOD model is better for modelling VEMs in the time domain with less computational cost. In this method (FOD), there is no need for extra dissipative coordinates. Therefore, the generalized degree of freedom of the system will not rise further [14, 17-20].

In this article, nonlinear transient analysis of the plate with active constrained layer (ACL) 0-3 VEC is done using FOD model (Figure 1). 0-3 VEC incorporates the graphite blocks/wafers in Butyl rubber matrix in a rectangular array [10]. Since damping performance of structural plate is dependent on several geometric parameters, such as the number of graphite blocks/wafers and viscoelastic thickness ratio within 0-3 VEC [10]. The transient responses of these parameters are analyzed under geometrically nonlinear vibrations. Further, hybrid damping of structural plate is estimated through the performance index at several geometric parameters of 0-3 VEC. And the optimal geometric parameters of 0-3 VEC are decided based on the utmost performance index. The results are concluded by comparing the damping performance of 0-3 VEC and conventional VEM to control the nonlinear vibrations of structural plate.

Materials and Methods

The structural plate is constructed with substrate layer/plate, constrained 0-3 VEC layer and active piezoelectric constraining layer (Figure 1). However, 0-3 VEC incorporates the graphite blocks/wafers in Butyl rubber matrix in a rectangular array [10]. The material properties for Aluminum substrate, graphite blocks/wafers and piezoelectric constraining layer are taken from reference [1, 10, 14]. For Butyl rubber VEM phase, Poisson's ratio v = 0.49 and mass density ρ = 920 kg/m^3, while the FOD parameters are taken as E_o = 9.0483 MPa, E_∞ = 194.1 MPa, τ = 5.55 μs and α = 0.84 [14].

In order to analyze the transient responses, initially, closed-loop FE modelling was done using von Kármán nonlinearity, layer wise-FSDT, and velocity feedback control strategy. However, the VEM phase within 0-3 VEC is modelled FOD method. The mathematical formulation of the overall structural plate is detailed in the adjacent section.

Mathematical formulation of structural plate

The schematic depiction of a plate with ACL 0-3 VEC is exhibited in figure 1. Correspondingly, h, h_d, and h_c indicate the thickness of substrate/base plate, constrained and active piezoelectric constraining layer. While a, b denotes the length and width of structural plate (Figure 1). 0-3 VEC damping layer consist by integrating graphite wafers within conventional VEM Butyl rubber matrix in a form of rectangular array [10]. In x/y axis, same quantity of graphite blocks/wafers (n_g) and space (Δ) between two consequent graphite blocks/wafers are considered. Moreover, top/bottom thickness (h_v) of VEM phase is also same within 0-3 VEC (Figure 1). In present analysis, uniformly distributed step load $p(t)$ is employed in transverse direction at the base of the plate (z = 0) while the clamped boundary conditions are considered at the edges of substrate plate (Figure 1). The structural plate is modelled in FE framework. Since structural plate consist of five thin layers so plane stress assumption is considered in the along the z-axis and the layer wise-FSDT is considered. Accordingly, for the k^{th} layer, the displacements coordinates (u^k, v^k, w^k) at any point along x, y and z axes are exhibited as:

$$u^k = u_0 + z^k\phi_x, v^k = v_0 + z^k\phi_y, w^k = w_0 \tag{1}$$

Since in the formulation geometric nonlinearity is taken into account by implementing von Kármán relation. Accordingly, state of strain/stress (ε/σ) for the k^{th} layer is exhibited as,

$$\varepsilon_b^k = \left\{\varepsilon_x^k\varepsilon_y^k\varepsilon_{xy}^k\right\}^{\mathrm{T}}, \varepsilon_s^k = \left\{\varepsilon_{xz}^k\varepsilon_{yz}^k\right\}^{\mathrm{T}}, \sigma_b^k = \left\{\sigma_x^k\sigma_y^k\sigma_{xy}^k\right\}^{\mathrm{T}}, \sigma_s^k = \left\{\sigma_{xz}^k\sigma_{yz}^k\right\}^{\mathrm{T}}; \text{ or}$$

$$\varepsilon_b^k = \left(\varepsilon_{bL} + \varepsilon_{bNL} + Z_b^k k_b\right), \varepsilon_b^k = \left(\varepsilon_{sL} + Z_s^k k_s\right) \tag{2}$$

Since structural plate consists of five layers. The substrate (k = 1) and constraining layer (k = 5) is made of isotropic material and piezoelectric material, respectively while constrained damping layer (k = 2, 3, 4) is made of 0-3 VEC. So, for isotropic material (k = 1) and piezoelectric material (k = 5), constitutive relations are expressed as in equations (3) and (4), respectively. The poling of piezoelectric layer is in z axis. It is actuated through the exterior voltage (V) enforced over electrode faces (top/bottom).

$$\sigma_b^k = C_b^k\varepsilon_b^k, \sigma_s^k = C_s^k\varepsilon_s^k \tag{3}$$

$$\sigma_b^k = C_b^k\varepsilon_b^k - e_b^k E_z, \sigma_s^k = C_s^k\varepsilon_s^k - D_z^k = \left(e_b^k\right)^{\mathrm{T}}\varepsilon_b^k + \in_{33}^k E_z, \quad \left(\text{where } E_z = -V/h_c\right) \tag{4}$$

Besides, the VEM phase within 0-3 VEC is modelled using FOD method. The explicit procedure for the modelling VEM phase using FOD constitutive relation are explained in [14, 17-20]. Although, the final expression of VEM phase in time domain using FOD parameters (E_0, E_∞, t, α) at any time step (n + 1)$^{\text{th}}$ is exhibited in equation (5).

$$\left(\boldsymbol{\sigma}_b^k\right)_{n+1} = \left\langle 1 - c\left(\frac{E_\infty - E_0}{E_0}\right)\right\rangle \boldsymbol{C}_b^k\left(\boldsymbol{\varepsilon}_b^k\right)_{n+1} + \left(\frac{cE_\infty}{E_0}\right)\boldsymbol{C}_b^k\sum_{j=1}^{N_t} A_{j+1}\left(\bar{\boldsymbol{\varepsilon}}_b^k\right)_{n+1-j}$$

$$\left(\boldsymbol{\sigma}_s^k\right)_{n+1} = \left\langle 1 - c\left(\frac{G_\infty - G_0}{G_0}\right)\right\rangle \boldsymbol{C}_s^k\left(\boldsymbol{\varepsilon}_s^k\right)_{n+1} + \left(\frac{cG_\infty}{G_0}\right)\boldsymbol{C}_s^k\sum_{j=1}^{N_t} A_{j+1}\left(\bar{\boldsymbol{\varepsilon}}_s^k\right)_{n+1-j}$$

Where, $c = \dfrac{\tau^\alpha}{\tau^\alpha + (\Delta t)^\alpha}$, $A_{j+1} = \dfrac{\Gamma(j-\alpha)}{\Gamma(-\alpha)\Gamma(j+1)}$ or $A_{j+1} = \dfrac{j-\alpha-1}{j}A_j$

(5)

Further, discretization of the reference plane of structural plate has been done by taking 9-node quadrilateral isoparametric elements for FE model. Since the layers within structural plate are made of substrate and VEM or graphite, and piezoelectric so the typical element is considered with different stacking arrangement [14]. So, the displacement vector (***d***) can expressed as ***Nd***^{e}, where ***N*** designates shape function vector. Substituting equations (2)-(5) in the variation of potential, kinetic energy (δT_p, δT_K in equation (6), (7)). The expression of δT_p, δT_K are further substituted in extended Hamilton's formula [14]. The global equation of motion is obtained by the assembly of elemental equations for time step (n + 1)th (equation (8)).

$$\delta T_p = \int_0^a\int_0^b\left[\sum_{k=1}^{5}\int_{h_k}^{h_{k+1}}\left\langle(\delta\boldsymbol{\varepsilon}_b^k)^{\mathrm{T}}\boldsymbol{\sigma}_b^k + (\delta\boldsymbol{\varepsilon}_s^k)^{\mathrm{T}}\boldsymbol{\sigma}_s^k\right\rangle dz - \int_{h_k}^{h_{k+1}}(\delta E_z)^{\mathrm{T}} D_z^k\Big|_{k=5} dz - \left\langle\delta w_0\, p(t)\right\rangle\Big|_{z=0}\right] dydx \tag{6}$$

$$\delta T_K = \int_0^a\int_0^b\left[\sum_{k=1}^{5}\int_{h_k}^{h_{k+1}}\left\langle\left\{\delta(\dot{u}^k)\quad \delta(\dot{v}^k)\quad \delta(\dot{w}^k)\right\}\rho^k\left\{(\dot{u}^k)\quad(\dot{v}^k)\quad(\dot{w}^k)\right\}^{\mathrm{T}}\right\rangle dz\right] dydx \tag{7}$$

$$\boldsymbol{M}\ddot{\boldsymbol{d}}_{n+1} + \left(\boldsymbol{C}_L + \boldsymbol{C}_{bNL}\right)\dot{\boldsymbol{d}}_{n+1} + \left(\boldsymbol{K}_L + \boldsymbol{K}_{bNL}\right)\boldsymbol{d}_{n+1} = \boldsymbol{P}_M\, p(t) - \left(\boldsymbol{P}_{vL} + \boldsymbol{P}_{vbN}\right) \tag{8}$$

Since equation (8) is a nonlinear transient equation, it is solved using the Newmark-beta method and direct iteration procedures [14, 18, 21]. However, in the present analysis, FE code is written, and its solution is estimated using MATLAB software package.

Results and Discussion

The nonlinear transient analysis of the plate with ACL 0-3 VEC is done in the section. The dimensions of structural plate are considered as a = 0.4 m, b = 0.4 m, h = 5 mm, h_d = 1 mm, h_c = 0.5 mm, Δ = 100 µm (Figure 1). However, clamped boundary condition is taken at the edges of substrate plate and uniformly distributed step load is applied in transverse direction at the base of the plate (z = 0) with intensity (p) 40 kN/m^2 (Figure 1) and the results are analysed at constant control-gain k_d = 100. Initially, the verification of the aforesaid FE formulation using FOD viscoelastic constitutive model is done. Figure 2 shows the present result obtained from the FE code for a constrained layer damping. Although, reference result is also presented in the same figure (Figure 2) obtained from literature [20]. While comparing, it may conclude that the results obtained from present FE code are excellent agreement with the reference results. Therefore, it confirms the correctness in modelling constrained layer damping using FOD method.

Since VEM thickness ratio (r_v = h_v/h_c) and number of graphite wafers (n_g) within 0-3 VEC layer are dependent on hybrid damping [10]. So, the influence of these geometric parameters (n_g, r_v) on nonlinear transient responses of the ACLD treated plate are analysed. Moreover, the effect of 0-3 VEC for ameliorated hybrid damping of structural plate is examined and compared with the conventional VEM without incorporating graphite wafers within VEM phase. However, the performance index $I_d = \left[\left\langle (W/h)_{t\,0} - (W/h)_{t\ 0.05\,\mathrm{s}}\right\rangle / (W/h)_{t\ 0}\right]$ represent the damping performance, where, W/h denotes the maximum transverse displacement-amplitude of nonlinear transient vibration of structural plate [14, 18].

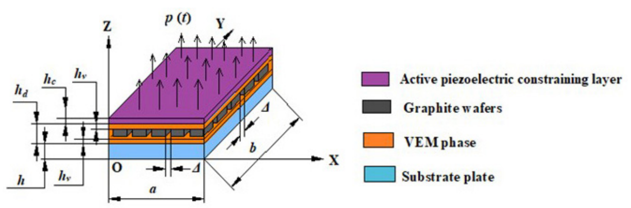

Figure 1: Schematic depiction of a plate with ACL 0 - 3 VEC.

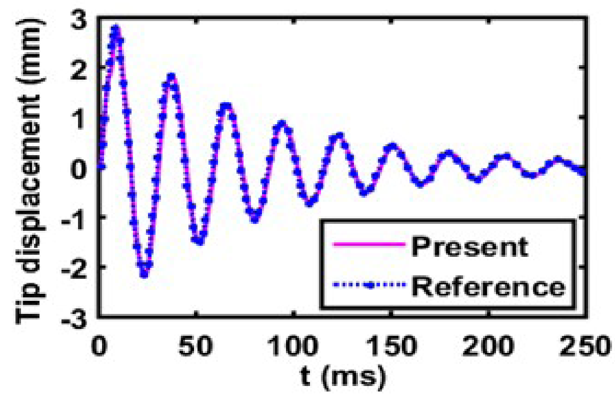

Figure 2: Comparison of present results obtained from FE code with reference results (Reference: Galucio et al. [20]).

Initially, the influence of parameters r_v and n_g on maximum vibration-amplitude (W/h) of the plate are studied. Figure 3a and 3b manifest the nonlinear transient plots of structural plate for several r_v at n_g = 6 and, several n_g at r_v = 0.12, consecutively. It may conclude through figure 3 that attenuation in maximum vibration-amplitude (W/h) varies with n_g and r_v of 0-3 VEC. So, the combined effect of these parameters (n_g, r_v) on damping performance of structural plate are examined explicitly. The performance index (I_d) is evaluated corresponding to several values of configured parameters (n_g, r_v) of 0-3 VEC within limits, 1 ≤ n_g ≤ 10 and 0.02 ≤ r_v ≤ 0.22. A 2-D grid is prepared within these limits and correspondingly performance index (I_d) is computed at every grid point. These results are presented through the contour as in figure 4. Figure 4 illustrates that the performance index (I_d) of the ACLD treated plate increases by incorporating graphite wafers (n_g) and VEM thickness ratio (r_v). However, maximum performance index (I_d) of the plate with active constrained 0-3 VEC layer is obtained at n_g = 4, r_v = 0.08 (point M, figure 4).

Further, the damping performance of the plate with ACL 0-3 VEC is compared with VEM without incorporating graphite wafers within the VEM phase. Figure 5a and 5b illustrate the nonlinear transient response and required control voltage of the structural plate at optimal geometric parameters (n_g = 4, r_v = 0.08) of 0-3 VEC, consecutively. In parallel, it (Figure 5) also shows the responses for VEM. Figure 5 illustrates that the attenuation in maximum vibration-amplitude (W/h) of structural plate is significantly more with less required control voltage for 0-3 VEC compared to VEM. The results indicate that the 0-3 VEC may be advantageous compared to the conventional VEM to control the geometrically nonlinear vibrations of the ACLD treated plate.

Conclusions

In this work, the hybrid damping control of nonlinear

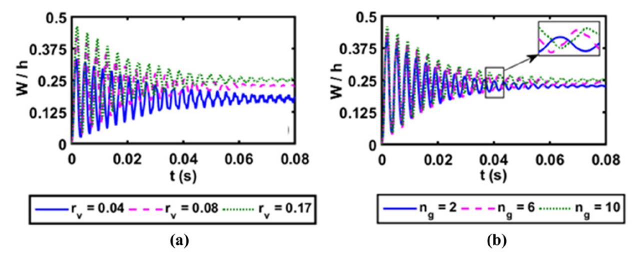

Figure 3: Transient response of ACLD treated plate at **(a)** several r_v and at n_g = 6 and **(b)** several n_g and at r_v = 0.12 within 0 - 3 VEC layer.

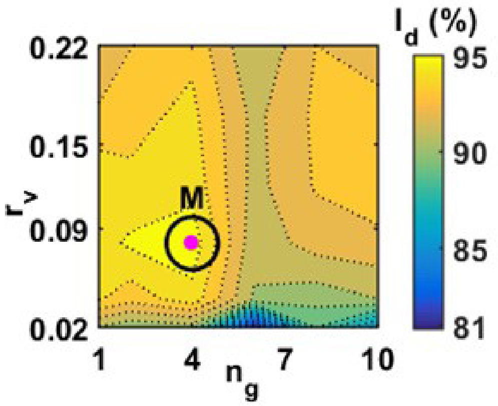

Figure 4: Variation of performance index (I_d) within two-dimensional domain of r_v and r_v within 0 - 3 VEC layer of ACLD treated plate.

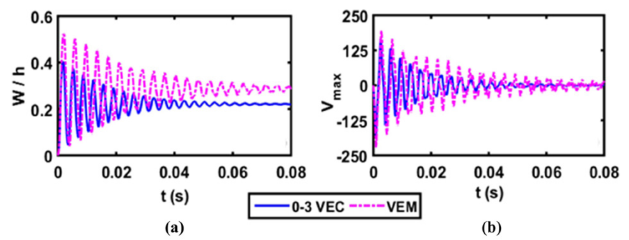

Figure 5: Transient response of the plate with ACL 0 - 3 VEC/VEM **(a)** vibration-amplitude and **(b)** control-voltage.

transient responses of plate with ACL 0-3 VEC is done. The structural plate is modelled using layer wise FSDT in the closed-loop FE framework. However, the VEM phase within 0-3 VEC is modelled using FOD method. Initially, the influence of graphite wafers and VEM thickness ratio on nonlinear transient responses of structural plate is estimated. It is found that the attenuation in the vibration-amplitude depends on these geometric parameters of 0-3 VEC. So, for maximum improvement in damping, the optimal geometric parameters of 0-3 VEC are obtained based on the utmost performance index of structural plate. It is found that the attenuation in vibration-amplitude is significantly more with less required control-voltage of structural plate for 0-3 VEC at optimal geometric parameters compared to conventional VEM. These observations reveal that the 0-3 VEC may be advantageous to control the nonlinear vibrations of ACLD treated plate.

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Conflict of Interest

The authors declare no conflict of interests that are relevant to the content of this article.

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Credit Author Statement

Abhay Gupta: Conceptualization, Methodology, Investigation, Formal analysis, Writing - original draft preparation, Supervision; Rajidi Shashidhar Reddy: Investigation, Formal analysis, Writing - original draft preparation; B.M. Girish: Resources, Writing - review and editing; Nitish Gupta: Resources, Writing - review and editing. All the authors read and approved the manuscript.

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