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Anisotropic Cosmological Model Bianchi type–III for Cloud String with Volume Viscosity in General Relativity

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Abstract

The anisotropic Cosmological model Bianchi type-III for cloud string with volume viscosity are studied, to get the determinate solution, the coefficient of volume viscosity assumed as a power function of an expansion $\xi = k\theta^n$ and shear scalar (σ) is directly proportional to scalar expansion (θ), which leads to the relation between metric potential $B = C^{\alpha}$. In the physical features is found that the power ends has a significant influence on the cloud string model, when n < 1 then 'big bang' starts and n > 1 then no big bang start, In the special condition n = 0 the model reduces to the string model of constant-coefficient of volume viscosity.

Keywords

Viscosity, Cosmology

Introduction

Knowing the precise model of the physical state at the beginning of the universe's development is still a challenge. As the universe's temperature dropped following the big bang, it appears that it may have gone through several phase transitions, expanding into a cosmos with density parameters larger than one and no entropy with a horizon [1]. If the rest mass of matter particles is constant, a universe with a time-varying gravitational "constant" G unavoidably entails creation [2]. Gravitational radiation from membranes will depend on the membrane topology [3, 4]. Due to the large-scale dispersion of stars and galaxies in our universe without energy conservation, the matter distribution is well characterized by a perfect fluid [5]. The distribution of matter must be considered, except for the perfect ideal fluid, as in the early stages of the cosmos, matter behaves like a viscous fluid. The cosmological constant's estimated value is still much at odds with observation, according to the energy of the vacuum [6]. Although it seems unlikely that there are many cosmic strings in the observable universe right now, their existence may have had a profound impact on the universe's early history [7]. Other scholars have extensively researched viscous fluid cosmological models in the early cosmos and concluded that the power law solutions reflect the characteristic of viscous solutions with variable G [8]. The current string Cosmological model of Bianchi type-III and the rotationally symmetric Bianchi cosmic cloud string with bulk viscosity. It is understood that the bulk viscosity coefficient is not constant, but rather decreases through time [9] and expands in the early cosmos. From the field equations, a first-order autonomous system is created, and the relationship between energy and momentum exhibits a line singularity [10], f(R) gravity is used to explain the general energy-momentum tensor, pressure, and density [11]. The exciting prospect of realizing the long-sought links between the many forms of particle interactions is heralded by several recent advances in particle physics [12]. While electromagnetic, strong, and weak interactions can all be combined.

In this letter, the anisotropic Cosmological model Bianchi type-III for cloud string with volume viscosity were studied. To obtain the determinate solution, the coefficient of volume viscosity assumed as a power function of an expansion $\xi = k\theta^n$ and shear scalar (σ) is directly proportional to scalar expansion (θ), which leads to the relation among matric potential $B = C^\alpha$. The physical and geometrical features related to the above model were also discussed.

Metric and Field Equation

We consider general Bianchi – III type space-time metric

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2\beta x}dy^2 + C^2dz^2 \qquad (2.1)$$

Where β is constant ($\beta > 0$) and A, B, C are functions of t-alone.

The tensor of energy-momentum for a cloud of string bulk viscosity is

$$T_{ij} = \rho\mu_i\mu_j - \lambda x_i x_j - \xi\theta(\mu_i\mu_j + g_{ij}) \qquad (2.2)$$

Where rest energy density ($\rho_p = \rho - \lambda$) and λ is the tension density of the cloud of the string θ is the scalar of expansion and ξ is the coefficient of bulk viscosity. The energy density for the coupled system ρ and λ is > 0 while the tension density λ > 0 or λ < 0. The symbol μ^i represents the velocity and x^μ represents a direction of anisotropy i.e. The direction of string, given by the standard relation.

$$\mu^i\mu_i = -x^ix_i = -1 \qquad \text{And} \qquad \mu^ix_i = 0 \qquad (2.3)$$

The expressions of kinematical parameters scalar of expansion and shear scalar are

$$\theta = \mu^i_j = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \qquad (2.4)$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}$$

$$\sigma^2 = \frac{1}{3}\left(\left(\frac{A_4}{A}\right)^2 + \left(\frac{B_4}{B}\right)^2 + \left(\frac{C_4}{C}\right)^2 - \frac{A_4B_4}{AB} - \frac{B_4C_4}{BC} - \frac{C_4A_4}{CA}\right) \qquad (2.5)$$

Einstein's equation we can consider here is

$$R_i^j - \frac{1}{2}Rg_i^j = -T_i^j + \Lambda g_i^j$$

$$R_{ij} - \frac{1}{2}Rg_{ij} = T_{ij} \qquad (2.6)$$

Where we have chosen the unit such that c = 1 and $8\pi G = 1$, In the co-moving coordinates $\mu^i = \delta_o^i$ and $\mu_i = -\delta_i^o$. With the help of equation (2.1) to (2.3), Einstein's equation (2.6) can be written as

$$\left[\frac{B_{44}}{B} + \frac{B_4C_4}{BC} + \frac{C_{44}}{C}\right] = \xi\theta \qquad (2.7)$$

$$\left[\frac{A_{44}}{A} + \frac{A_4C_4}{AC} + \frac{C_{44}}{C}\right] = \xi\theta \qquad (2.8)$$

$$\left[\frac{A_{44}}{A} + \frac{B_4A_4}{BA} + \frac{B_{44}}{B} - \frac{\beta^2}{A^2}\right] = \lambda + \xi\theta \qquad (2.9)$$

$$\left[\frac{A_4B_4}{AB} + \frac{C_4A_4}{CA} + \frac{B_4C_4}{BC} - \frac{\beta^2}{A^2}\right] = \rho \qquad (2.10)$$

$$\left.\begin{aligned}\beta\left(\frac{B_4}{B} - \frac{A_4}{A}\right) = 0\\ \frac{B_4}{B} - \frac{A_4}{A} = 0\end{aligned}\right\} \qquad (2.11)$$

Where A_4 denotes the differentiation of A with respect to time t.

The solution of field equations

From the equation (2.11), we can have

$$\beta\left(\frac{B_4}{B} - \frac{A_4}{A}\right) = 0 \Rightarrow \frac{B_4}{B} - \frac{A_4}{A} = 0 \qquad (3.1)$$

Where b being a constant of integration

To obtain a more general solution, we assume

$$\xi = k\theta^n \qquad (3.2)$$

Where 'k' and 'n' are positive constants

We note that the five independent equations (2.7) to (2.11) and (3.2) connect six unknown variables (A, B, C, λ, σ and ξ)

Thus, one more relation connecting these variables in needed to solve these equations.

In order to obtain explicit solutions, one additional relation is needed, and use adopt an assumption that the shear scalar is proportional to the scalar of expansion [13].

$\sigma \alpha \theta$ which leads to

$$B = C^\alpha \qquad (3.3)$$

Where 'n' is constant

Substitution equation (3.3) into equation (2.4) and using equation (3.2) we have

The expansion is required [13] and it is given by

$$\theta = \mu^i_j = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} +$$

$$\theta = (2\alpha + 1)\frac{C_4}{C} \qquad (3.4)$$

$$\xi\theta = K^*\frac{C_4^{\,n+1}}{C^{n+1}} \qquad (3.5)$$

Where $K^* = k(2\alpha + 1)^{n+1}$

With the help of equation (3.3) and (3.5), the equation reduced to

$$\frac{C_{44}}{C}+\frac{\alpha^2}{\alpha+1}.\left(\frac{C_4}{C}\right)^2=\frac{K}{\alpha+1}.\frac{C_4^{\,n+1}}{C^{n+1}} \qquad (3.6)$$

To solve the equation (3.6), we denote $C_4=\eta$ then

$$C_{44}=\eta\frac{d\eta}{dC} \qquad (3.7)$$

Equation (3.6) leads to

$$\frac{d\eta}{dC}+\frac{\alpha^2}{\alpha+1}.\frac{\eta}{C}=\frac{K^*}{\alpha+1}.\frac{\eta^m}{C^m} \qquad (3.8)$$

Let $$N=\frac{\alpha^2}{\alpha+1} \qquad (3.9)$$

$$\frac{d\eta}{dC}+N\frac{\eta}{C}=\frac{K^*}{\alpha+1}.\frac{\eta^m}{C^m} \qquad (3.10)$$

Equation (3.10) rewritten as

$$\frac{d}{dx}\left(\eta^{1-n}.C^{(1-n)N}\right)=\frac{(1-n)K}{(\alpha+1)}.C^{(1-n)N-n} \qquad (3.11)$$

A Solution of the equation easily be obtained

$$\eta=C_4=\left[\frac{K^*C^{1-n}}{(N+1)(\alpha+1)}+MC^{(n-1)N}\right]^{\frac{1}{1-n}} \qquad (3.12)$$

We have $N=\dfrac{n^2}{n+1}$

$$\eta=C_4=\left[\frac{K^*C^{1-n}}{\alpha^2+\alpha+1}+MC^{(n-1)N}\right]^{\frac{1}{1-n}} \qquad (3.13)$$

With the help of equation (3.13) equation (2.1) reduce to

$$ds^2=-\left[\frac{K^*C^{1-n}}{(\alpha^2+\alpha+1)}+MC^{(n-1)N}\right]^{\frac{-2}{1-n}}dC^2+b^2C^{2\alpha}dx^2+C^{2\alpha}e^{2\beta x}dy^2+C^2dz^2 \qquad (3.14)$$

Under suitable transformation of co-ordinate, equation (3.14) reduce to

$$ds^2=-\left[\frac{K^*T^{1-n}}{(\alpha^2+\alpha+1)}+MT^{(\alpha-1)N}\right]^{\frac{-2}{1-n}}dT^2+b^2T^{2\alpha}dx^2+T^{2\alpha}e^{2\beta x}dy^2+T^2dz^2 \qquad (3.15)$$

Physical and Geometrical Properties of the Model

Equation (3.15) represent our model. The expression of physical and geometrical properties given by

$$\rho=\alpha(\alpha+2)\left[\frac{K}{(\alpha^2+\alpha+1)}+MT^{(n-1)(N+1)}\right]^{\frac{-2}{1-n}}-\frac{\beta^2T^{2\alpha}}{b^2} \qquad (4.1)$$

$$\lambda=\left(\frac{\alpha-1}{1-n}\right)\left[\frac{K^*T^{1-n}}{(\alpha^2+\alpha+1)}+MT^{(n-1)N}\right]^{\frac{1+n}{1-n}}.\left[\frac{(1-n)K^*T^{-(1+n)}}{(\alpha^2+\alpha+1)}+M(n-1)NT^{(n-1)N-2}\right]$$
$$+2\alpha(\alpha-1)\left[\frac{K^*}{(\alpha^2+\alpha+1)}+MT^{(n-1)(N+1)}\right]^{\frac{2}{1-n}}-\frac{\beta^2T^{-2n}}{b^2} \qquad (4.2)$$

$$\rho_p=\frac{1-\alpha}{1-n}\left[\frac{K^*T^{1-n}}{(\alpha^2+\alpha+1)}+MT^{(n-1)N}\right]^{\frac{1+n}{1-n}}.\left[\frac{(1-n)K^*T^{-(1+n)}}{(\alpha^2+\alpha+1)}+M(n-1)NT^{(n-1)N-2}\right]$$
$$+\alpha(4-\alpha)\left[\frac{K}{(\alpha^2+\alpha+1)}+MT^{(n-1)(N+1)}\right]^{\frac{2}{1-n}} \qquad (4.3)$$

$$\theta=(2\alpha+1)\left[\frac{K}{(\alpha^2+\alpha+1)}+MT^{(n-1)(N+1)}\right]^{\frac{1}{1-n}} \qquad (4.4)$$

$$\sigma^2=\frac{(\alpha-1)^2}{3}\left[\frac{K^*}{(\alpha^2+\alpha+1)}+MT^{(n-1)(N+1)}\right]^{\frac{2}{1-n}} \qquad (4.5)$$

From equation (4.1) it was observed that the realistic condition $\rho\geq 0$ was fulfilled when

$$\alpha(\alpha+2)\left[\frac{K^*}{(\alpha^2+\alpha+1)}+MT^{(n-1)(N+1)}\right]^{\frac{-2}{1-n}}\geq\frac{\beta^2T^{-2n}}{b^2}$$

Case – I (n < 1)

In the presence of Bulk viscosity scalar of expansion θ tends to infinity large and the energy density $\rho\to\infty$ when $T\to 0$ but θ tends to finite and ρ tends to finite when in the absence of bulk viscosity K = 0, $\theta\to\infty$ and $\rho\to 0$ when $T\to\infty$. Hence the model was non-rotating expanding shearing universe.

Case – II (n > 1)

In the presence of bulk viscosity θ tends to zero when $T\to\infty$ but θ tends to be finite when $T\to 0$. And in the absence of bulk viscosity K = 0, $\theta\to\infty$ when $T\to 0$. Hence $\rho\to\infty$ when $T\to 0$ and $\rho\to 0$ when $T\to\infty$. Therefore the model describes a shearing non-rotating expanding universe with viscosity [14], without the big-bang start we can see from the above discussion that the bulk viscosity plays a significant role in the evolution of the universe since $\lim\limits_{T\to\infty}\frac{\sigma}{\theta}\neq 0$ and T is large than model does not approach isotropy , The shear scalar σ > 0 when n = 1 , hence n = 1 is the isotropy condition.

Case – III (n = 0)

$$ds^2=-\left[\frac{K^*T}{(\alpha^2+\alpha+1)}+MT^{-N}\right]^{-2}dT^2+b^2T^{2\alpha}dx^2+T^{2\alpha}e^{2\beta x}dy^2+T^2dz^2 \qquad (4.6)$$

$$\aleph = \quad \ddot{u} \quad + \quad \left[\frac{K^{\ddot{u}}}{\ddot{u}\alpha^2 + \alpha +} + MT^{-\ddot{u}N+} \quad \right]^2 - \frac{\beta \; T^{- \;\; \alpha}}{b^2} \tag{4.7}$$

$$\lambda = \left[\frac{(2\alpha^2 - \alpha - 1)K^*}{(\alpha^2 + \alpha + 1)} + \frac{\alpha(\alpha+2)(\alpha-1)MT^{\aleph(N \;\; 1)}}{\alpha + 1} \right] \cdot \left[\frac{K^*}{(\alpha^2+\alpha+1)} + MT^{-(N+1)} \right] - \frac{\beta^2 T^{\;\; 2\alpha}}{b^2} \tag{4.8}$$

$$\rho_p = \left[\frac{(1+3\alpha-\alpha^2)K}{(\alpha^2+\alpha+1)} + \frac{2\alpha(\alpha+2)MT^{-(N+1)}}{\alpha+1} \right] \cdot \left[\frac{K^*}{(\alpha^2+\alpha+1)} + MT^{-(N+1)} \right] \tag{4.9}$$

$$\theta \; \aleph(2\alpha \quad 1) \left[\frac{K^*}{(\alpha^2+\alpha+1)} \quad MT^{-1(N+1)} \right] \tag{4.10}$$

$$\sigma^2 = \frac{(\alpha-1)^2}{3} \left[\frac{K^*}{(\alpha^2+\alpha+1)} + MT^{-1(N+1)} \right]^2 \tag{4.11}$$

In the absence of bulk viscosity K = 0, the model reduces to the string model without viscosity that is

$$ds^2 = -M^{-2}T^{2N}dT^2 + b^2T^{2\aleph}dx^2 + T^2 \;\; e^{2 \;\; x}dy^2 + T^2dz^2 \tag{4.12}$$

$$\aleph\aleph \quad (\qquad 2)M^2T^{-2(N+1)} \qquad \frac{\beta^2T^{-2\alpha}}{b^2} \tag{4.13}$$

$$\lambda = \left[\frac{\alpha(\alpha+2)(\alpha-1)\ddot{u} \;\; ^{2} \quad ^{\aleph 2(N \;\; 1)}}{\alpha+1} \right] - \frac{\beta^2 \quad ^{2n}}{b^2} \tag{4.14}$$

$$\rho_p = \left[\frac{2\alpha(\alpha+2)M^2T^{-2(N+1)}}{\alpha+1} \right] \tag{4.15}$$

$$\theta = (2\alpha+1)\left[MT^{-1(N+1)} \right] \tag{4.16}$$

$$\sigma^2 = \frac{(\alpha-1)^2}{3}\left[M^2T^{-2(N+1)} \right] \tag{4.17}$$

From the equation, it observed that the realistic conditions was fulfilled when

$$\frac{\alpha(\alpha+2)M^2}{T^{2(N+1)}} \geq \frac{T^{-2\alpha}}{b^2}$$

The scalar of expansion $T \to 0$ when $T \to 0$ and $\theta \to 0$ when $T \to \infty$ provided $n > -\frac{1}{2}$ and the scalar of expansion in the model was monotonically increasing when $0 < T < \infty$ since $\lim_{T \to \infty} \frac{\sigma}{\theta} \neq 0$ the model does not approach isotropy for a large value of T, However the energy density $\rho \to \infty$ when $T \to 0$ and $\rho \to 0$ when $T \to \infty$ therefore the model describes a continuously expanding shearing non-rotating universe with the big-bang start

Case – IV (special case n = 1)

$$\frac{d\eta}{dC} + \beta^* \frac{\eta}{C} = 0 \tag{4.18}$$

Where $\beta^* = \frac{\alpha^2 - K^*}{\alpha + 1}$

Integrating of the equation gives $\eta = \frac{M}{C^{\beta^*}}$

When M is the constant of integration In the same way as performed above, we can easily obtain

$$ds^2 = -M^{-2}T^{2\beta^*}dT^2 + b^2T^{2\alpha}dx^2 + T^{2\alpha}e^{2\beta x}dy^2 + T^2dz^2 \tag{4.19}$$

$$\rho = \alpha(\alpha+2)M^2T^{-2(N+1)} - \frac{\beta^2T^{-2\alpha}}{b^2} \tag{4.20}$$

$$\lambda = (2\alpha - \beta^*)(\alpha-1)M^2T^{-2(\beta^*+1)} - \frac{\beta^2T^{-2n}}{b^2} \tag{4.21}$$

$$\rho_p = [\alpha(4-\alpha) + (\alpha-1)\beta^*]M^2T^{-2(\beta^*+1)} \tag{4.22}$$

$$\theta = (2\alpha+1)\left[MT^{-1(\beta^*+1)} \right] \tag{4.23}$$

$$\sigma^2 = \frac{(\alpha-1)^2}{3}\left[M^2T^{-2(\beta^*+1)} \right] \tag{4.24}$$

It was clear that the realistic condition $\rho \geq 0$ be fulfilled when

$$\frac{\alpha(\alpha+2)M^2}{T^{2(\beta^*+1)}} \geq \frac{\beta^2T^{2n}}{b^2}$$

The scalar of expansion θ is infinitely large at T = 0 and θ tends to zero when $T \to \infty$ provide $\beta^* + 1 > 0$, Hence the model was non-rotating shearing and expanding universe with the big-bang. The energy density $\rho \to \infty$ when $T \to 0$ ρ tends to zero and $T \to \infty$ Hence the model represents the shearing and non-rotating expanding universe with the big-bang start.

Conclusions

We have studied the anisotropic Cosmological model Bianchi Type–III for cloud string with volume viscosity to be able to obtain a more general modal we assume that the coefficient of bulk viscosity is a power function of the scalar of expansion $\xi = k\theta^n$ and ration of shear scalar and expansion in not equal to zero [15], so the shear scalar is proportional to the scalar of expansion $\aleph$. The physical feature of the model is also discussed; It was found that the power index m has a significant influence on the string modal. There is a big-bang start in the model when $n \leq 1$ but there is not any big bang start when n > 1. When n = 0 the model reduces to the string model of the constant coefficient of bulk viscosity.

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