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Mixed Mode (I/II) Fracture Studies in PMMA Using Single Edge Cracked Circular Specimen

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Abstract

Most often, the engineering components under service loads develop a crack in arbitrary orientation, which may result in fracture under mixed mode conditions. In this study, a recently proposed single edge cracked circular (SECC) specimen is used for performing mixed mode (I/II) fracture studies of polymethylmethacrylate (PMMA). The SECC specimens fabricated from the PMMA sheet have been used to conduct fracture experiments under static loading. A large number of finite element (FE) numerical analyses of the SECC specimen have been performed to determine the stress intensity factors and *T*-stress for different mixed mode configurations. The critical fracture load and crack kink angles have been measured from fractured specimens and are compared with the generalized maximum tangential stress (GMTS) criterion. The experimental results are in good agreement with the GMTS criterion. The results show that the newly proposed SECC specimen can successfully be utilized to study the mixed mode fracture behavior of brittle materials like PMMA.

Keywords

Stress intensity factor, Mixed mode fracture, Polymethylmethacrylate, Finite element, Experiment

Introduction

The PMMA is frequently used as a favorite brittle material for conducting experimental investigations of crack growth. Some major advantages of PMMA include optical transparency, ease of specimen fabrication, and, more importantly, ease of creating sharp precrack. Due to the advantages mentioned above, PMMA has been widely employed by various researchers in fracture studies [1-3]. The engineering components are often subjected to multi-axial loading conditions. Under service loads, these structures develop cracks like defects, slowly propagating and resulting in fractures. Therefore, mixed mode fracture studies are essential for predicting structural integrity and ensuring its reliability. A crack under mixed mode loading grows in a curvilinear manner. Therefore, a complete knowledge of mixed mode (I/II) fracture needs to calculate (a) the critical load P_{cr} at which unstable crack propagation occurs and (b) the direction of incipient crack propagation θ_0.

Due to the lack of standardized testing procedures, a large number of laboratory specimens have been employed by various researchers to study the mixed mode fracture behavior of various brittle materials like rocks, glass, and metals. Among them, compact tension shear specimens [4], Arcan specimen [5], T-specimen [6], semi-circular bend specimens [7], and three/four-point bend specimens [8] have been widely used for mixed mode fracture studies. Shukla et al. [9] have recently proposed a SECC specimen that can be used for mixed mode fracture studies. The SECC specimen has demonstrated its efficacy in conducting fracture

studies of brittle and metallic materials. The main advantage of the SECC specimen is that the specimen does not require any complicated additional fixture to impart mixed mode loading.

In the present work, new fracture tests under mixed mode loading have been conducted on the SECC specimens fabricated from the PMMA material. The mixed mode fracture toughness of the PMMA is evaluated. Detailed FE analyses have also been performed to calculate the variation of stress intensity factors and *T*-stress. Finally, the experimental results are compared with the theoretical predictions of maximum tangential stress (MTS) and GMTS criterion.

Materials and Methods

Design of the single edge cracked circular specimen

The schematic diagram of the SECC specimen is shown in figure 1. The SECC specimen has a diameter equal to D_o = 160 mm. The specimen has three holes of diameter d = 13 mm. Three holes are strategically positioned to serve two purposes. First, if the specimen is fabricated from a metallic specimen, holes A and C can be used for the fatigue pre-cracking. Second, holes A and B are used to load the SECC specimen under mixed mode conditions. By changing the position of holes, A and B with respect to the loading line, desired mode mixity can be achieved. The angle between the crack plane and the global loading direction is defined as β. And the angle $\alpha = (\pi/2) - \beta$ is defined as the loading angle (Figure 1b). The case of mode I can be achieved at a loading angle $\alpha = 0^\circ$, and the case of mixed mode (I/II) can be achieved at a loading angle $\alpha \neq 0^\circ$. Further details about the design and development of the SECC specimen can be read from Shukla et al. [9].

Theoretical background

In the context of linear elastic fracture mechanics, various stress, strain, and energy-based mixed mode (I/II) fracture criteria have been developed to predict the unstable crack growth direction and critical fracture load [10-12]. The mathematical formulation of MTS [10] and GMTS criteria [12] have been briefly presented in this section.

According to the MTS criterion [10], the crack grows in the plane perpendicular to the maximum tangential stress when this stress reaches a critical value K_{IC}. The crack initiation angle θ_0 can be calculated from equation 1.

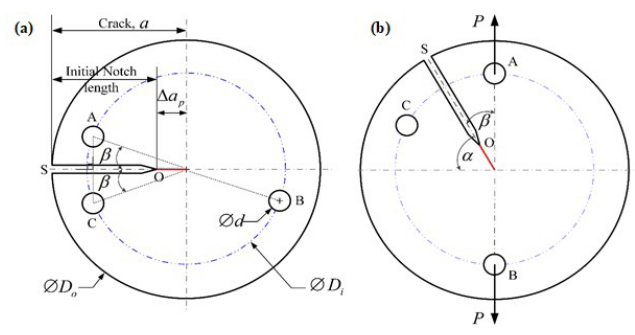


Figure 1: Geometric details and loading orientation of the SECC specimen. **(a)** Precracking orientation and **(b)** Mixed mode configuration.

$$\theta_0 = 2\tan^{-1}\left[\frac{1}{4}\frac{K_I}{K_{II}} - \frac{1}{4}\sqrt{\left(\frac{K_I}{K_{II}}\right)^2 + 8}\right] \quad \text{for} \quad K_{II} > 0$$
$$\theta_0 = 2\tan^{-1}\left[\frac{1}{4}\frac{K_I}{K_{II}} + \frac{1}{4}\sqrt{\left(\frac{K_I}{K_{II}}\right)^2 + 8}\right] \quad \text{for} \quad K_{II} < 0 \qquad (1)$$

Where, K_I and K_{II} are the mode I and mode II stress intensity factors (SIFs), respectively. The fracture locus of the MTS criterion under mixed mode loading can be calculated from equation 2.

$$\frac{K_I}{K_{IC}} = \left[\cos\frac{\theta_0}{2}\left(\cos^2\frac{\theta_0}{2} - \frac{3}{2}\frac{K_{II}}{K_I}\sin\theta_0\right)\right]^{-1}$$
$$\frac{K_{II}}{K_{IC}} = \left[\cos\frac{\theta_0}{2}\left(\frac{K_I}{K_{II}}\cos^2\frac{\theta_0}{2} - \frac{3}{2}\sin\theta_0\right)\right]^{-1} \qquad (2)$$

Here, K_{IC} is the plane strain fracture toughness.

The GMTS criterion considers the singular SIF terms (K) and first non-singular term known as *T*- stress near the crack tip for the calculation of tangential stress [12]. Other assumptions of the GMTS criterion are the same as the MTS criterion [10]. According to the GMTS criterion, the crack initiation angle θ_0 can be calculated from equation 3

$$K_I \sin\theta_0 + K_{II}(3\cos\theta_0 - 1) - \frac{16T\sqrt{2\pi r_c}}{3}\sin\frac{\theta_0}{2}\cos\theta_0 = 0 \qquad (3)$$

Where, T is the non-singular term *T*-stress and r_c is defined as the size of the localized damage zone of the material [12]. The fracture locus of the GMTS criterion can be calculated from equation 4. In the present investigation r_c = 0.3 mm is considered for PMMA material, which is in the range of values used by previous studies on PMMA [1-3, 12, 13].

$$\frac{K_I}{K_{IC}} = \left[\cos\frac{\theta_0}{2}\left(\cos^2\frac{\theta_0}{2} - \frac{3}{2}\frac{K_{II}}{K_I}\sin\theta_0\right) + \sqrt{2\pi r_c}\sin^2\theta_0\frac{T}{K_I}\right]^{-1}$$
$$\frac{K_{II}}{K_{IC}} = \left[\cos\frac{\theta_0}{2}\left(\frac{K_I}{K_{II}}\cos^2\frac{\theta_0}{2} - \frac{3}{2}\sin\theta_0\right) + \sqrt{2\pi r_c}\sin^2\theta_0\frac{T}{K_{II}}\right]^{-1} \qquad (4)$$

It is to be noted from equations 3 and 4 that the GMTS criterion simplifies to the MTS criterion if the *T*-stress term is not considered.

Finite element analyses

The FE technique has been used to perform all the numerical simulations in the ANSYS® APDL environment. Since the loading is planar, two-dimensional analysis of the specimen has been performed under plane stress assumptions. The following dimensions of the SECC specimen have been considered for FE analyses: Do = 160 mm, Di = 118 mm, d = 13 mm, and different crack lengths. The material properties of the PMMA, Young's modulus E = 2950 MPa and Poisson's ratio ϑ = 0.38 are used in all the FE analyses. Figure 2 shows the typical FE mesh of the SECC specimen used for the present work. Eight-noded-isoparametric quadratic elements have been used to generate the mesh of the SECC specimen.

Twenty-four quarter-point elements (QPEs) have been used circumferentially. QPEs are generally used for simulating the inverse square root singularity near the crack tip. The boundary conditions used for the numerical analyses are shown in figure 2. A reference point load of P = 1000 N is applied on hole A, which has also been constrained in the x-direction. All degrees of freedom of hole B have been kept equal to zero (Figure 2).

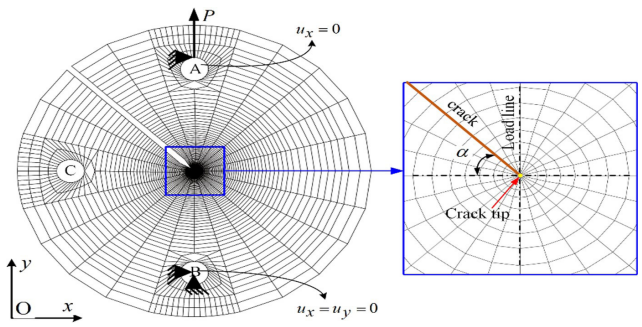


Figure 2: Left-hand side image shows the FE mesh of the SECC specimen. The right-hand side image shows the spider web arrangement of QPEs around the crack.

The SIFs and T-stress are calculated using an interior collocation method proposed by Jogdand and Murthy [14]. The required computer codes for implementing the interior collocation method have been written in MATLAB. Since the SIFs characterize the crack tip stress fields, the accurate determination of SIFs and T-stress plays an important role in the theoretical predictions using a suitable fracture criterion. In the present analyses, the variation of SIFs and T-stress are obtained by varying the loading angle α from 0° to 80° for different crack length ratios a/D_o. For simplicity, the SIFs and T-stress for any mixed mode configuration of the SECC specimen can be written as given in equation 5.

$$K_I = \frac{P}{D_o t}\sqrt{\frac{\pi a}{2}}Y_I\left(\frac{a}{D_o},\alpha\right); \quad K_{II} = \frac{P}{D_o t}\sqrt{\frac{\pi a}{2}}Y_{II}\left(\frac{a}{D_o},\alpha\right); \quad T = \frac{P}{D_o t}T^*\left(\frac{a}{D_o},\alpha\right) \tag{5}$$

Where, P is the applied load, t is the thickness of the SECC specimen, a is the crack length, and $Y_I(a/D_o,\alpha)$ and $Y_{II}(a/D_o,\alpha)$ are the mode I and mode II geometric factors, respectively. And T^* is the dimensionless form of the T-stress.

The variation of geometric factors $Y_I(a/D_o,\alpha)$, $Y_{II}(a/D_o,\alpha)$, and T^* for different crack length ratios $0.4 \le a/D_o \le 0.7$ are shown in figure 3. It can be observed from figure 3a that Y_I monotonically decreases with an increase in α. On the other hand, Y_{II} increases with the increase in loading angle as expected. For, α = 0°, $Y_I \neq 0$°, Y_{II} = 0° which implies that the SECC specimen is under pure mode I loading. For $\alpha \neq 0$°, both Y_I and Y_{II} are non-zero, which indicates that the specimen is under mixed mode (I/II) loading conditions. As expected, with an increase in crack length from a/D_o = 0.4 to 0.7, the magnitude of both Y_I and Y_{II} increases. Figure 3c shows the variation T^* for four different crack length ratios. It is evident from Figure 3c that the sign of T^* changes from negative to positive as the loading angle α changes from 0° to 80°. For a/D_o = 0.7, T^* remains positive for the complete range of loading angles $0° \le \alpha \le 80°$. It is to be noted here that the sign and magnitude of the T-stress considerably affect the fracture toughness and crack path under mixed mode loading [12]. A positive T-stress decreases the fracture toughness and a negative T-stress increases the fracture toughness of cracked materials. Furthermore, a negative T-stress assists in stable crack growth [12, 13].

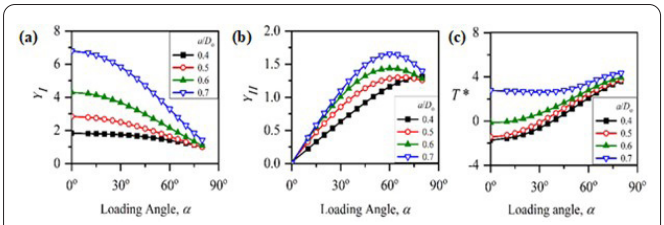


Figure 3: (a) Variation of mode I geometric factor, **(b)** Variation of mode II geometric factor, and **(c)** Variation of normalized T-stress for various crack length ratios.

Fracture experiments

Mixed mode fracture experiments for six different loading angles, i.e., α = 0°, 15°, 30°, 45°, 60°, and 75° have been conducted on the SECC specimen fabricated from a 6 mm thick PMMA sheet material. The specimens are fabricated using a CO_2 laser cutting machine, and a precrack of length 2 mm is cut with the help of a jewelry saw, making a final crack length ratio of a/D_o = 0.5. A BiSS ITW make 250 kN servo hydraulic universal testing machine has been used for all the fracture experiments. Figure 4 shows the complete experimental loading setup used for fracture experiments under mixed mode loading. All the tests are conducted under a constant displacement rate of 0.15 mm/min.

Results and Discussion

Table 1 shows the experimental results for the SECC specimen tested under different mixed mode loading angles. The mode I fracture toughness of the PMMA sheet is calculated by considering the SIFs value corresponding to fracture load for α = 0° (Table 1). The average K_{IC} of the SECC speci-

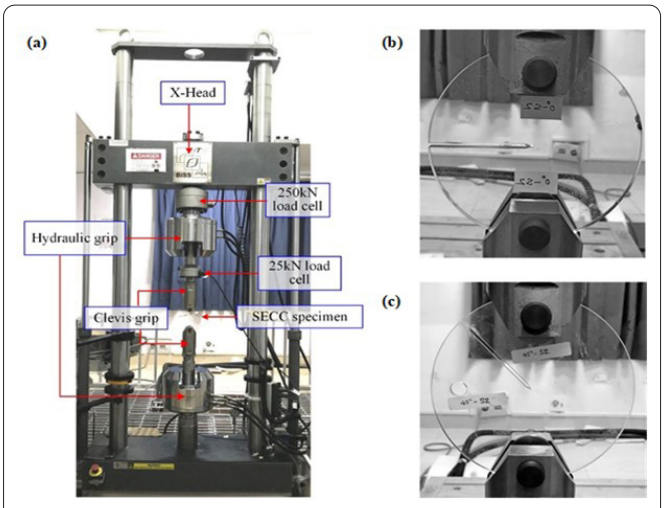


Figure 4: (a) Complete experimental setup for mixed mode fracture tests, **(b)** Pure mode I, and **(c)** Mixed mode (I/II) loading configuration for α = 45°.

Table 1: Experimental results of the fracture tests on PMMA using the SECC specimen.

Specimen ID	α (°)	P_{cr} (N)	K_{If} (MPa) $\sqrt{mm}$	K_{IIf} (MPa $\sqrt{mm}$)	θ_0^{avg} (°)
0-S1	0	1086	51.0	0	0
0-S2		1153	54.2	0	0
0-S3		1139	53.5	0	0
15-S1	15	1155	52.3	9.0	21
15-S2		1117	50.6	8.7	20
15-S3		1115	50.5	8.7	21.5
30-S1	30	1174	48.3	16.5	31.5
30-S2		1262	51.9	17.8	31.2
30-S3		1176	48.4	16.6	30.4
45-S1	45	883	30.7	16.5	40.4
45-S2		1020	35.4	19.1	39.5
45-S3		1246	43.3	23.3	40.5
60-S1	60	937	25.3	19.8	50.4
60-S2		1099	29.7	23.3	53.1
60-S3		1103	29.8	23.3	52.3
75-S1	75	1473	27.9	31.2	59.5
75-S2		1483	28.1	31.5	60
75-S3		1265	24.0	26.8	58.3

mens is found to be equal to K_{IC} = 52.9 MPa $\sqrt{mm}$. The K_{IC} for the tested PMMA material is in the range of 32 - 63 MPa $\sqrt{mm}$ which has also been reported by various researchers [1-3, 12, 13]. The measured crack initiation angles θ_0^{avg} of the SECC specimens are compared with the predictions of MTS and GMTS criteria (Figure 5a). It can be seen from figure 5a that the experimental results agree well with the GMTS criterion for r_c = 0.3 mm. The fracture limit curves of the tested PMMA material, along with the MTS and GMTS criteria, are shown in figure 5b. It is evident from figure 5b that the experimental data has a natural scatter. And the average of experimental results (shown as a square legend in red color figure 5b) agrees well with the predictions of the GMTS criterion.

Conclusion

The mixed mode (I/II) fracture properties of the PMMA material has been investigated using a new set specimen named single edge cracked circular (SECC). The mode I fracture toughness of the PMMA material is found to be equal to K_{IC} = 52.9 MPa $\sqrt{mm}$. The theoretical prediction of the GMTS criterion for r_c = 0 are in good agreement with the experimental results.

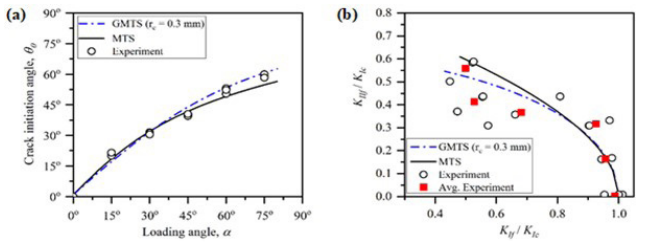


Figure 5: (a) Comparison of experimental crack initiation angle and **(b)** Mixed mode fracture locus for PMMA with the theoretical prediction of GMTS criterion.

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Conflict of Interest

The authors declare that they have no known conflict of interests that could have appeared to influence the work reported in this paper.

Credit Author Statement

Shiv Sahaya Shukla: Conceptualization, Methodology, Formal Analysis, Data curation, Investigation, Writing - original draft preparation, Writing - review and editing; K.S.R. Krishna Murthy: Conceptualization, Methodology, Investigation, Visualization, Project administration, Resources, Supervision. All the authors read and approved the manuscript.

References

1. Aliha MRM, Bahmani A, Akhondi S. 2016. Mixed mode fracture toughness testing of PMMA with different three-point bend type specimens. *Eur J Mech A Solids* 58: 148-162. https://doi.org/10.1016/j.euromechsol.2016.01.012

2. Ayatollah, MR, Aliha MRM, Hassani MM. 2006. Mixed mode brittle fracture in PMMA—an experimental study using SCB specimens. *Mater Sci Eng A* 417(1-2): 348-356. https://doi.org/10.1016/j.msea.2005.11.002
3. Aliha MRM, Ayatollahi MR. 2010. Geometry effects on fracture behaviour of polymethyl methacrylate. *Mater Sci Eng A* 527(3): 526-530. https://doi.org/10.1016/j.msea.2009.08.055
4. Richard HA, Benitz K. 1983. A loading device for the creation of mixed mode in fracture mechanics. *Int J Fract* 22: R55-R58. https://doi.org/10.1007/BF00942726
5. Banks-Sills L, Arcan M, Bortman Y. 1984. A mixed mode fracture specimen for mode II dominant deformation. *Eng Fract Mech* 20(1): 145-157. https://doi.org/10.1016/0013-7944(84)90122-X
6. Demir O, Ayhan AO, İriç S. 2017. A new specimen for mixed mode-I/II fracture tests: modeling, experiments and criteria development. *Eng Fract Mech* 178: 457-476. https://doi.org/10.1016/j.engfracmech.2017.02.019
7. Aliha MRM, Ayatollahi MR. 2013. Two-parameter fracture analysis of SCB rock specimen under mixed mode loading. *Eng Fract Mech* 103: 115-123. https://doi.org/10.1016/j.engfracmech.2012.09.021
8. Razavi SMJ, Aliha MRM, Berto F. 2018. Application of an average strain energy density criterion to obtain the mixed mode fracture load of granite rock tested with the cracked asymmetric four-point bend specimens. *Theor Appl Fract Mech* 97: 419-425. https://doi.org/10.1016/j.tafmec.2017.07.004
9. Shukla SS, Sajith S, Murthy KSRK. 2022. A new simple specimen for mixed-mode (I/II) fracture and fatigue tests: numerical and experimental studies. *Eur J Mech A Solids* 94: 104566. https://doi.org/10.1016/j.euromechsol.2022.104566
10. Erdogan F, Sih GC. 1963. On the crack extension in plates under plane loading and transverse shear. *J Basic Eng* 85(4): 519-525. https://doi.org/10.1115/1.3656897
11. Maiti SK, Smith RA. 1984. Criteria for brittle fracture in biaxial tension. *Eng Fract Mech* 19(5): 793-804. https://doi.org/10.1016/0013-7944(84)90162-0
12. Smith DJ, Ayatollahi MR, Pavier MJ. 2001. The role of T-stress in brittle fracture for linear elastic materials under mixed-mode loading. *Fatigue Fract Eng Mater Struct* 24(2): 137-150. https://doi.org/10.1046/j.1460-2695.2001.00377.x
13. Ayatollahi MR, Aliha MRM, Saghafi H. 2011. An improved semi-circular bend specimen for investigating mixed mode brittle fracture. *Eng Fract Mech* 78(1): 110-123. https://doi.org/10.1016/j.engfracmech.2010.10.001
14. Jogdand PV, Murthy KSRK. 2010. A finite element based interior collocation method for the computation of stress intensity factors and T-stresses. *Eng Fract Mech* 77(7): 1116-1127. https://doi.org/10.1016/j.engfracmech.2010.03.002