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Static and Free Vibration Properties of Functionally Graded Thin Plates Incorporating a Single Edge Crack and Under Uniaxial Static Loading Conditions

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Abstract

The current study performs static and free vibration analysis of a functionally graded (FG) plate made of Aluminium-Silicon Carbide (Al SiC) with an edge crack. A material gradation based on a linear power-law is considered along the plate thickness. The effects of changing the percentage of SiC gradation and varying crack angles on deformation, stress intensity factor (K), and natural frequencies are investigated. It is observed that with increase in percentage gradation of SiC in FG plate, the values of K and natural frequency increase but the deformation decreases. The results of the FG plate are validated using analytical and experimental results from the literature and compared to those of an isotropic Aluminum alloy Al 2014-T6 plate. It is also observed that FG plates have lower deformation and K values than Aluminum alloy Al 2014-T6 plates, making them suitable for a wide range of applications

Keywords

Functionally graded plate, Edge crack, Static, Free vibration, Finite element analysis

Introduction

Functionally graded materials (FGM) are composites that have continuous variation in material properties from one surface to the next. Thermal stresses, residual stresses, and stress concentrations are reduced as the material's properties are graded. Furthermore, tailored material response and exceptional thermal performance can be achieved with FGM [1, 2]. As a result of advancements in computational and manufacturing capabilities over the past few decades, extensive research on FGM has been done.

One of the first papers on FG plates was by Reddy [3] in which, theoretical and numerical formulations based on third order shear deformation plate theory were presented. Reddy and Cheng [4] also proposed the three dimensional (3D) thermomechanical deformations of FG rectangular plates. Chi and Chung [5, 6] in their papers investigated the analytical and finite element analysis (FEA) solution of simply supported FG plates under transverse loadings. Bernardo et al. [7] carried out static and free vibration analysis of FG plates. Moleiro et al. [8] proposed a mixed layer wise model to carry out static analysis of FG plates under transverse mechanical loads. The literature on FGM is extensive and therefore, only selective papers are discussed in the present work. For more information, the reader may refer to detailed reviews by Jha et al. [9], Gupta and Talha [10], Thai and Kim [11], Zhang et al. [12], and Sinha and Kumar [13].

According to the literature review, FGM has found widespread applications in the industry for reducing stress concentrations and providing thermal shielding. It is also noted that very few works exist in literature, numerically analyzing

the static and free vibration properties of FG plates in the presence of single edge cracks. Hence, in the current study, an FG plate incorporating an edge crack is considered. A linear power-law based material gradation along the plate thickness is considered. The effects of change in percentage of gradation, varying crack angles, and loading amplitudes are studied and the results obtained from an FG plate are compared to those from an Aluminum alloy plate. Geometric design, material gradation, convergence, and validation study are discussed in the following section.

Experimentation

Theory and validation

Displacement, strain, and stress are redistributed around the crack tip when a crack exists in a structure. The displacement and stress fields around the crack tip in mode I are expressed as [14]:

$$\left.\begin{aligned}
\sigma_{xx} &= \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1-\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]\\
\sigma_{yy} &= \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1+\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]\\
\tau_{xy} &= \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1-\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right)\right]\\
u_x &= \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\cos\left(\frac{\theta}{2}\right)\left[K-1+2\sin^2\left(\frac{\theta}{2}\right)\right]\\
u_y &= \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\sin\left(\frac{\theta}{2}\right)\left[K+1-2\cos^2\left(\frac{\theta}{2}\right)\right]
\end{aligned}\right\} \quad (1)$$

Where, σ_{xx} and σ_{yy} are the normal stresses, τ_{xy} is the shear stress, and u_x and u_y are the displacement components. K_I is the stress intensity factor for mode I configuration, r is the radial distance and θ is the orientation angle around the crack tip. v is the Poisson's ratio, μ is the shear modulus, k is 3 - 4v for plane strain and $\frac{3-v}{1+v}$ for plane stress, respectively.

The parameters affecting the stress intensity factor (K) are far field stress (σ), crack size (a), geometry and mode of loading [14-17]. K values can be determined via analytical closed form solutions, experimental methods, and numerical simulations. However, analytical solutions exist only for simple geometries and material selection [14-16], and experimental methods are inefficient in terms of cost. As a result, numerical methods, in particular FEA have emerged as the preferred method for analyzing a wide range of geometry and material selection. In the current study, a thin rectangular plate with an edge crack as shown in figure 1 is considered for analysis.

The plate with a horizontal edge crack is loaded in a mode I configuration with a uniaxial tensile distributed loading on the edge BC with a magnitude of 7500 N and fixed on the bottom edge AD. Crack growth in the plate is influenced by its fracture toughness (K_c) and K_I. Crack will propagate under static loading conditions if:

$$K_I \geq K_C \quad (2)$$

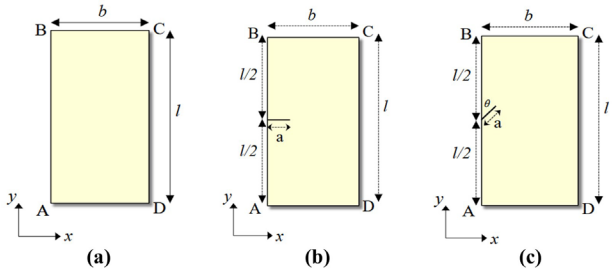


Figure 1: Geometric configuration of thin rectangular plate with and without crack. **(a)** No crack, **(b)** Horizontal edge crack, and **(c)** Angled edge crack.

Where, K_I for mode I is given as [14]:

$$K_I = \beta\sigma_C\sqrt{\pi a} \quad (3)$$

Here, the value of β is dependent on θ and is equal to 1.12 for a horizontal edge crack, a is the crack length and σ_c is the critical stress. The current study considers an isotropic material, Aluminum alloy (Al 2014-T6), and FGM Al SiC for analysis. The material properties for Al 2014-T6 are given in table 1.

The material properties for FGM are obtained using the linear power law function and rule of mixtures [5, 6] as given in Eq. 4 and 5, respectively. Young's modulus (E) and Poisson's ratio (v) are varied along the thickness direction.

Table 1: Material properties for Al 2014-T6 [8].

Material	Al2014-T6
Young's modulus	72.4 GPa
Poisson's ratio	0.33
Fracture toughness	19.5 MPa $\sqrt{m}$
Load magnitude	7.5 kN

Power-law function

$$f(t) = \left(\frac{t + h/2}{h}\right)^n \quad (4)$$

Where, $f(t)$ is the power-law function, h is the plate thickness and n is the material parameter wherein, for linear gradation value of n is considered as 1.

Rule of mixture:

$$E(t) = E_1 f(t) + (1 - f(t))E_2 \quad (5)$$

Where, $E(t)$ is the material property of each layer. E_1 and E_2 are the Young's moduli of the FG plate at the top surface at $t = h/2$ and the bottom surface at $t = -h/2$, respectively.

Selection of FGM layers

To determine the minimum number of layers required in an FG plate, a convergence study is performed. Layers range from 2 to 20. When compared to the analytical results [18],

figure 2 shows that the deformation remains unchanged after 10 layers. As a result, the current study includes 10 layers for FG plates. Figure 3 shows the corresponding converged mesh density with 40000 nodes and the FG layers. The material properties for Al SiC are shown in table 2.

Linear gradation from Al to SiC is performed using the power law function and rule of mixtures as given in Eqs. (3) and (4), respectively. The material properties are linearly graded, with the bottom layer consisting of 100% Al and the top layer consisting of 100%, 72%, 50%, and 25% SiC for the respective cases. The material properties for various percentages

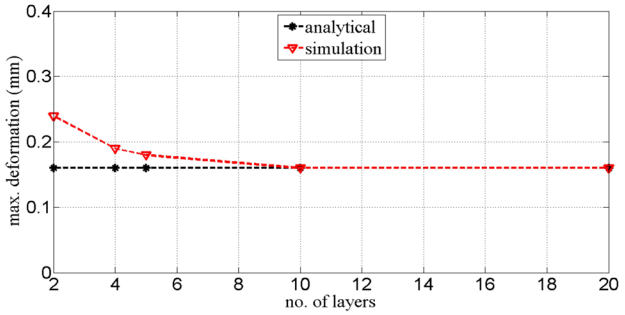


Figure 2: Convergence study to determine minimum number of FG layers.

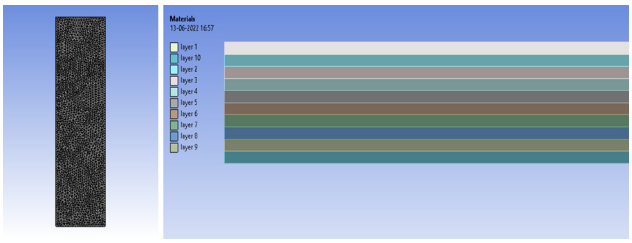


Figure 3: Mesh density and layers of FG plate.

of FGM gradations (SiC % on top layer) are shown in table 3.

Validation study

FEA-based solver ANSYS is used for simulations in the present study. A pre-meshed smart crack approach is adopted in which, crack's top, front, and bottom regions are characterized and coupled with node sets which are distributed around them. The sphere of influence procedure is then used to refine the mesh around the crack tip. An Al 2014-T6 plate with a horizontal and an angled edge crack, as shown in figure 1, are considered for analysis as part of the validation study. The geometric dimensions taken into account are as follows: length l = 180 mm, breadth b = 40 mm, thickness t = 2.5 mm, and angle θ = 40°. The crack lengths for horizontal edge crack and angled edge cracks are maintained at 9.3 mm and 8.6 mm respectively. A uniaxial tensile distributed load of magnitude of 7500 N is applied on the edge BC and the bottom edge AD is fixed. The results of FEA simulations are compared to those found in the literature [17] and shown in table 4. The simulation results for K_I are found to be in good agreement with the analytical and experimental results.

Table 2: Material properties for FGM Al SiC [].

Material	Al	SiC
Young's modulus	70 GPa	427 GPa
Poisson's ratio	0.3	0.17
Density (ρ)	2700 kg/m^3	3210 kg/m^3

The effects of changing the % of SiC gradation in FGM, and varying crack angle on static and modal analysis are studied in the following section.

Results and Discussion

Effect of varying crack angle

A parametric study is performed by varying the crack angle θ for Al 2014-T6 and FG plates. The % gradation of SiC on top layer of FG plate is kept at 25%. It is observed from figure 4 that as θ increases from 20° to 90°, K_I values increase. But as θ is increased beyond 90° to 160°, K_I values decrease. Thus, the maximum K_I values are observed at the horizontal edge crack. Furthermore, when compared to an isotropic Al

Table 3: Material properties for 10 FGM layers and various % of FGM gradations.

100 % gradation		**72 % gradation**		**50 % gradation**		**25 % gradation**	
***E* (GPa)**	***v***	***E* (GPa)**	***v***	***E* (GPa)**	***v***	***E* (GPa)**	***v***
427	0.17	317.1	0.21	230.65	0.24	150.32	0.27
391.3	0.183	289.69	0.22	212.8	0.24	141.4	0.27
355.6	0.196	262.23	0.23	194.95	0.25	132.47	0.27
319.9	0.209	234.76	0.24	177.1	0.26	123.55	0.28
284.2	0.222	207.30	0.25	159.25	0.26	114.62	0.28
212.8	0.248	179.84	0.26	141.4	0.27	105.7	0.28
177.1	0.261	152.38	0.27	123.55	0.28	96.77	0.29
141.4	0.274	124.92	0.28	105.7	0.28	87.85	0.29
105.7	0.287	97.46	0.29	87.85	0.29	78.92	0.29
70	0.3	70	0.3	70	0.3	70	0.3

Table 4: K_I values (MPa $\sqrt{mm}$) for Al 2014-T6 plate.

	Simulation	Analytical [15]	Experimental [17]
Horizontal edge crack	473.17	454.74	487
Angled edge crack	248.39	253.30	-

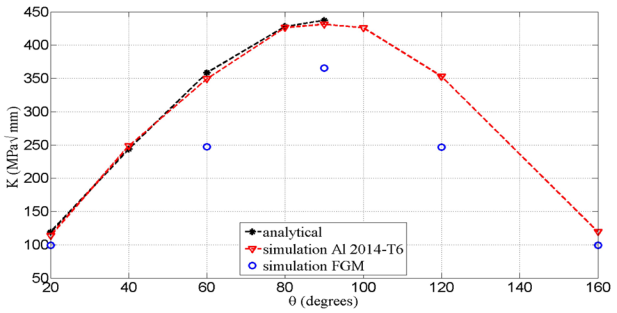


Figure 4: Effect of varying crack angle on K_I values.

2014-T6 plate, the magnitudes of K_I values generated by the FG plate are lower.

Effect of varying FGM gradation

Figure 5 and figure 7 show that as the % of SiC gradation in FG plate with horizontal edge crack increases, so does K_I, but deformation decreases by 38.1% to 73.3% when compared to Al 2014-T6 plate. Figure 6 and figure 8 show the K_I and deformation contours respectively, obtained by varying the % of SiC. Furthermore, when compared to the Al 2014-T6 plate, the natural frequency obtained for the FG plate was higher for all the modes (1-6) as shown in figure 9, which contributed to its lower deformation.

Conclusion

The effects of changing the SiC % in FG plates and varying crack angles are investigated in this paper. The results obtained from an FG plate are compared with those obtained from an isotropic Aluminum alloy plate (Al 2014-T6). The maximum value of K_I is observed at the horizontal edge crack. At different crack angles, K_I around the crack tip of FG plate is found to be lower than that of Al 2014-T6. K_I values around the crack tip for FGM plate are also found to be lower by 5% to 19.67%. As the gradation of SiC % in FG plate increases, K_I increases but deformation decreases by 38.1% to 73.3% when compared with Al 2014-T6 plate. Furthermore, when compared to Al

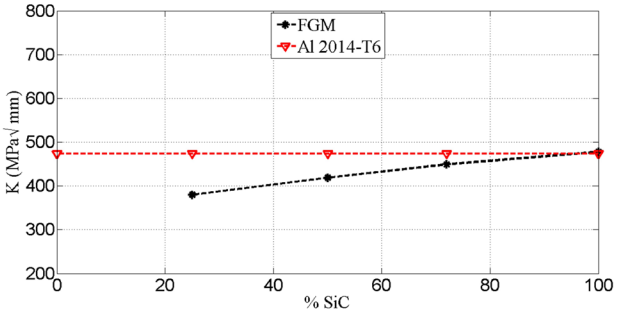


Figure 5: Effect of varying % of SiC gradation on K_I values.

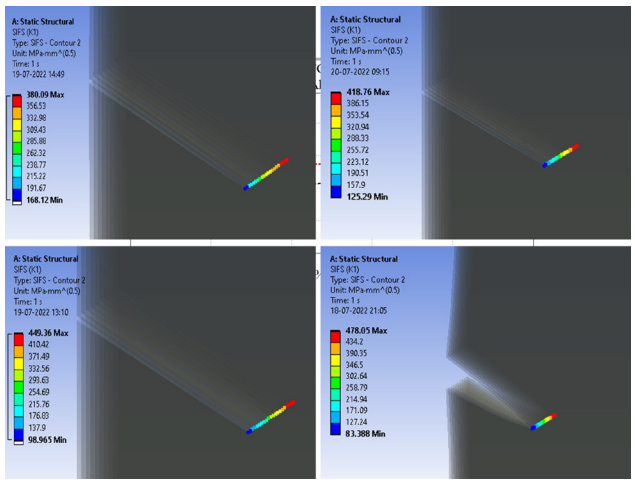


Figure 6: K_I contours obtained by varying % of SiC gradation.

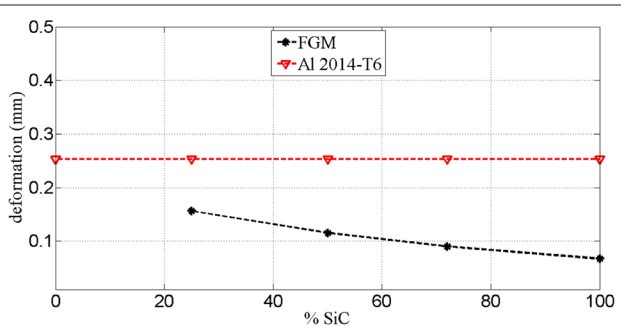


Figure 7. Effect of varying % of SiC gradation on maximum deformation.

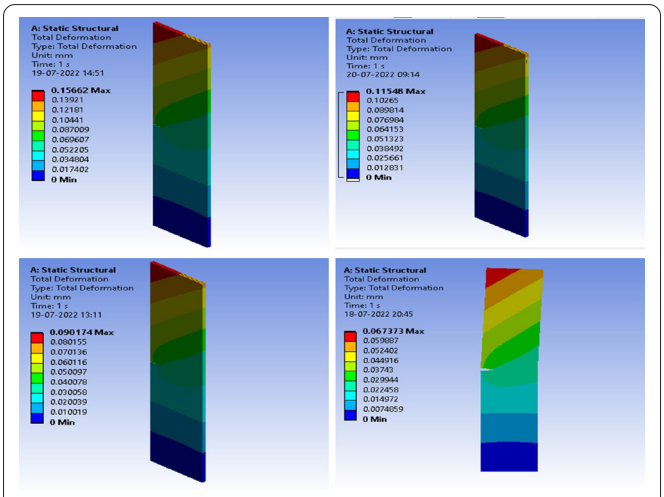


Figure 8: Deformation contours obtained by varying % of SiC gradation.

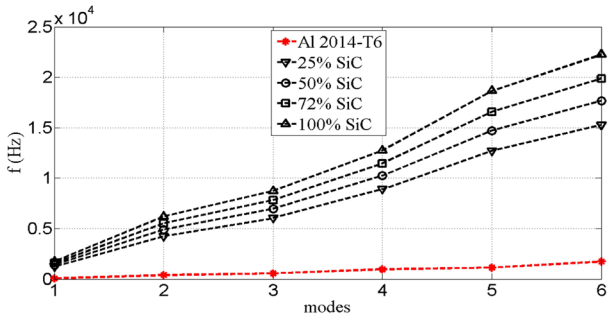


Figure 9: Effect of varying % of SiC gradation on natural frequencies.

2014-T6 plate, the natural frequency obtained for the FG plate (by varying SiC %) was higher, contributing to its lower deformation. Based on the results it can be asserted that with increase in gradation of SiC % deformation decreases but K_I and natural frequency values increase, thereby necessitating an optimum selection of SiC % based on applications.

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None.

Conflict of Interest

The authors declare no conflict of interests that are relevant to the content of this article.

Credit Author Statement

Gowtham H: Conceptualization, Methodology, Formal analysis, Investigation, Writing - original draft preparation, Resources; Shashank V: Conceptualization, Methodology, Writing - original draft preparation, Writing - review and editing, Resources, Supervision. All the authors read and approved the manuscript.

References

1. Cooley WG. 2005. Application of functionally graded materials in aircraft structures. Department of Aeronautics and Astronautics, Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio, US (Thesis).
2. Reddy JN. 2003. Mechanics of Laminated Composite Plates and Shells. CRC Press, Boca Raton.
3. Reddy JN. 2000. Analysis of functionally graded plates. *Int J Numer Methods Eng* 47(1-3): 663-684. https://doi.org/10.1002/(SICI)1097-0207(20000110/30)47:1/3<663::AID-NME787>3.0.CO;2-8
4. Reddy JN, Cheng ZQ. 2001. Three-dimensional thermomechanical deformations of functionally graded rectangular plates. *Eur J Mech A Solids* 20(5): 841-855. https://doi.org/10.1016/S0997-7538(01)01174-3
5. Chi SH, Chung YL. 2006. Mechanical behavior of functionally graded material plates under transverse load-Part I: analysis. *Int J Solids Struct* 43(13): 3657-3674. https://doi.org/10.1016/j.ijsolstr.2005.04.011
6. Chi SH, Chung YL. 2006. Mechanical behavior of functionally graded material plates under transverse load-Part II: numerical results. *Int J Solids Struct* 43(13): 3675-3691. https://doi.org/10.1016/j.ijsolstr.2005.04.010
7. Bernardo GMS, Damásio FR, Silva TAN, Loja MAR. 2016. A study on the structural behaviour of FGM plates static and free vibrations analyses. *Compos Struct* 136: 124-138. https://doi.org/10.1016/j.compstruct.2015.09.027
8. Moleiro F, Correia VF, Araújo AL, Soares CM, Ferreira AJM, et al. 2019. Deformations and stresses of multilayered plates with embedded functionally graded material layers using a layerwise mixed model. *Compos B Eng* 156: 274-291. https://doi.org/10.1016/j.compositesb.2018.08.095
9. Jha DK, Kant T, Singh RK. 2013. A critical review of recent research on functionally graded plates. *Compos Struct* 96: 833-849. https://doi.org/10.1016/j.compstruct.2012.09.001
10. Gupta A, Talha M. 2015. Recent development in modeling and analysis of functionally graded materials and structures. *Prog Aerosp Sci* 79: 1-14. https://doi.org/10.1016/j.paerosci.2015.07.001
11. Thai HT, Kim SE. 2015. A review of theories for the modeling and analysis of functionally graded plates and shells. *Compos Struct* 128: 70-86. https://doi.org/10.1016/j.compstruct.2015.03.010
12. Zhang N, Khan T, Guo H, Shi S, Zhong W, et al. 2019. Functionally graded materials: an overview of stability, buckling, and free vibration analysis. *Adv Mater Sci Eng* 2019: 1354150. https://doi.org/10.1155/2019/1354150
13. Sinha GP, Kumar B. 2021. Review on vibration analysis of functionally graded material structural components with cracks. *J Vib Eng Technol* 9: 23-49. https://doi.org/10.1007/s42417-020-00208-3
14. Anderson TL. 2017. Fracture Mechanics: Fundamentals and Applications. CRC Press, Boca Raton.
15. Kumar P. 2008. Elements of Fracture Mechanics. Tata McGraw-Hill.
16. Murakami Y. 1987. Stress Intensity Factors Handbook. Pergamon.
17. Tom J. 2014. Fracture study of edge cracked aluminium panel using extended finite element method. Department of Mechanical and Aerospace Engineering, Indian Institute of Technology Hyderabad, Telangana, India (Thesis).
18. Timoshenko S. 2002. Strength of Materials, 3E Vol. I: Elementary Theory and Problems. CBS Publishers.