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New Bound State Energies for Spherical Quantum Dots in Presence of a Confining Potential Model at Nano and Plank's Scales

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Abstract

In present work, the exact analytical bound-state solutions of modified Schrödinger equation with Modified central potential consisting of a Cornell-modified plus pseudoharmonic harmonic potential (MCMpH) have been presented using both Boopp's shift method and standard perturbation theory, we have also constructed the corresponding noncommutative Hamiltonian which containing two new terms, the first one is modified Zeeman effect and the second is new spin-orbital interaction. The theoretical results show that the automatically appearance for both spin-orbital interaction and modified Zeeman Effect leads to the degenerate to energy levels to $2(2l+1)$ sub states.

Keywords

Schrödinger equation, Star product, Boopp's shift method, Pseudoharmonic, linear, Coulomb potentials, Noncommutative space and noncommutative phase

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Introduction

It is well known that to study any quantum chemical-physical model, in different fields of sciences like atomic, nuclear, molecular, harmonic and harmonic spectroscopy, we need to solve the non relativistic Schrödinger equation and relativistic two equations: Klein-Gordon and Dirac [1-22]. To obtain profound interpretation in Nano and plank's scales, much work in case of the noncommutative space-phase at two, three and N generalized dimensions has been done for solving the three fundamental previously equations [23-47]. The notions of noncommutativity of space and phase developed on based to the Seiberg-Witten map, Boopp's shift method and the star product, defined on the first order of two infinitesimal parameters antisymmetric $\left(\theta^{\mu\nu},\bar{\theta}^{\mu\nu}\right)\equiv\frac{1}{2}\varepsilon^{ijk}\left(\theta_k,\bar{\theta}_k\right)$ as [23-47]:

$$f(x)*g(x)=f(x)g(x)-\frac{i}{2}\theta^{\mu\nu}\partial^x{}_\mu f(x)\partial^x{}_\nu g(x)-\frac{i}{2}\bar{\theta}^{\mu\nu}\partial^p_\nu f(x)\partial^p_\nu g(x) \quad \ldots\ldots(1)$$

Which allow us to obtaining the two new non nulls commutators $\left[\hat{x}_i,\hat{x}_j\right]_*$ and $\left[\hat{p}_i,\hat{p}_j\right]_*$, respectively as:

$$\left[\hat{x}_i,\hat{x}_j\right]_*=i\theta_{ij} \text{ and } \left[\hat{p}_i,\hat{p}_j\right]=i\bar{\theta}_{ij} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

It is important to notice, that the Boopp's shift method will be applying in this paper instead of solving the (NC-3D: RSP) with star product, the Schrödinger equation will be treated by using directly usual commutators on quantum mechanics, in addition to the two commutators [29-43]:

$$\left[\hat{x}_i,\hat{x}_j\right]=i\theta_{ij} \text{ and } \left[\hat{p}_i,\hat{p}_j\right]=i\overline{\theta}_{ij} \quad \text{.................................(3)}$$

The main goal of this work is to extend our study in reference [41] for the potential (MCMpH) including new term $\left(-\frac{d}{r^2}\right)$ into noncommutative three dimensional spaces and phases on based to the principal reference [48] to discover the new spectrum and possibility to obtain new applications for the modified potential in different fields. The rest of present search is organized as follows: In next section, we give briefly review to the Schrödinger equation with (CMpH) in three dimensional spaces. In section 3, we shall briefly introduce the fundamental concepts of Boopp's shift method and then we apply this method to derive the deformed potential and noncommutative spin-orbital Hamiltonian. In section 4, we apply perturbation theory to find the spectrum for ground stat and first excited states and then we deduce the spectrum produced automatically by the external magnetic field. In section 5, we conclude the global noncommutative Hamiltonian and we resume the global spectrum for (MCMpH) in first order of two infinitesimal parameters' Θ and $\overline{\theta}$ in (NC-3D: RSP). Finally, the important found results and the conclusions are discussed in the last section.

Review of the Eigenvalues and Eigenfunctions for (CMpH) in Three Dimensional

In this section, we shall review the eigenvalues and eignenfunctions for spherically symmetric for the potential known by Cornell-modified plus pseudoharmonic potential (CMpH) in three dimensional (3D) spaces [48]:

$$V(r)=ar^2+br-\frac{c}{r}-\frac{d}{r^2} \quad \text{..............................(4)}$$

The four parameters: $a\rangle$, b , c and d are constants, the above confining interaction potential consisting of a sum of harmonic, linear, Coulombic and pseudoharmonic potential terms, the last term is incorporated into the quarkouniom potential for the sake of coherence while the rest terms represents the Cornell potential [48, 49], the complex eigenfunctions $\Psi(r,\theta,\varphi)$ in 3-dimensional space for above potential satisfied the Schrödinger equation (SE) in spherical coordinates ($\hbar=c=1$) is [48]:

$$\left(-\frac{\Delta}{2m_0}+ar^2+br-\frac{c}{r}-\frac{d}{r^2}\right)\text{ø}(r,\theta,\varphi)=E_{n,l}\text{ø}(r,\theta,\varphi) \quad \text{.......(5)}$$

Where m_0 is the isotropic effective mass and $E_{n,l}$ is the total energy of the particle and $\Psi(r,\theta,\varphi)$ can be written as [48]:

$$\Psi(r,\theta,\varphi)=\sum_{n,l}N_l\psi_{n,l}(r)Y_l^m(\theta,\varphi) \quad \text{.........................(6)}$$

Here $Y_l^m(\theta,\varphi)$ is the spherical harmonic and the radial wave function $\psi_{n,l}(r)$ is the solution of the equation [48]:

$$\left(\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}-\frac{l(l+1)}{r^2}\right)\psi_{n,l}(r)+2m_0\left[E_{n,l}-ar^2-br+\frac{c}{r}+\frac{d}{r^2}\right]\psi_{n,l}(r)=0 \quad \text{...(7)}$$

To eliminate the first order of derivation, it's covariant to rewritten the radial wave function $\psi_{n,l}(r)$ to the form [48]:

$$\psi_{n,l}(r)=\frac{1}{r}\phi_{n,l}(r) \quad \text{...(8)}$$

Then, the equation (7) reduces to following form [48]:

$$\frac{d^2\phi_{n,l}(r)}{dr^2}+\left[\varepsilon_{n,l}-a_1r^2-b_1r+\frac{c_1}{r}+\frac{d}{r^2}+\frac{d_1-l(l+1)}{r^2}\right]\phi_{n,l}(r)=0 \quad \text{.........(9)}$$

Where, $(\varepsilon_{n,l},a_1,b_1,c_1,d_1)=2m_0(E_{n,l},a,b,c,d)$ and then, the complete normalized wave functions and corresponding energies for the ground state, the first existed states, and n^{th} excited state, respectively [48]:

$$\Psi^{(0)}(\vec{r})=N_{0l}r^{(l'-1)/2}\exp\left(-\tfrac{1}{2}\sqrt{2m_0a}r^2-\frac{2m_0c}{l'+1}r\right)Y_l^0(\theta,\varphi)$$

$$E_{0l}=\sqrt{\frac{a}{2m_0}}(3+l')-\frac{2m_0c^2}{(l'+1)^2} \quad \text{.....(10.1)}$$

$$\psi^{(1)}(\vec{r})=N_{1l}\left(r-\alpha_1^{(1)}\right)^{(l'-1)/2}\exp\left(-\frac{1}{2}\sqrt{2m_0ar^2}-\sqrt{\frac{m_0}{2a}}br\right)Y_l^0(\theta,\varphi)$$

$$E_{1l}=\sqrt{\frac{a}{2m^0}}(4+l')-\frac{b^2}{4a} \quad \text{....(10.2)}$$

$$\psi^{(l)}(\vec{r})=N_l\prod_{i=1}^{n}\left(r-\alpha_i^{(n)}\right)^{(l'-1)/2}\exp\left(-\frac{1}{2}\sqrt{2m_0ar^2}-\sqrt{\frac{m_0}{2\alpha}}br\right)Y_l^m(\theta,\varphi)$$

$$E_{nl}=\sqrt{\frac{a}{2m_0}}(2+2n+l')-\frac{b^2}{4a} \quad \text{....(10.3)}$$

Where $l'=\sqrt{(2l+1)^2-8m_0d}$ and the two normalized constants (N_{0l} , N_{1l}) are given by [48]:

$$N_{0l}=\frac{1}{\sqrt{\Gamma(l')D_{-l'}\left(\frac{4m_0c}{l'+1}\sqrt{\frac{1}{2m_0\omega}}\right)}}(2m_0\omega)^{l'/4}\exp\left(-\frac{m_0c^2}{4\omega(l'+1)^2}\right)$$

$$N_{1l}=\frac{(2m_0\omega)^{l'/4}\exp\left(-\frac{b^2}{4m_0\omega^3}\right)}{\sqrt{\frac{1}{2m_0\omega}\Gamma(l'+2)S_1+\alpha_1^{(1)2}\Gamma(l')S_2-2\alpha_1^{(1)}\sqrt{\frac{1}{2m_0}}\Gamma(l'+1)S_3}} \quad \text{....(11)}$$

With $S_1=D_{-(l'+2)}\left(\sqrt{\frac{2}{m\omega}}\frac{b}{\omega}\right)$, $S_2=D_{-(l')}\left(\sqrt{\frac{2}{m\omega}}\frac{b}{\omega}\right)$, $S_3=D_{-(l'+1)}\left(\sqrt{\frac{2}{m_0\omega}}\frac{b}{\omega}\right)$,

$\alpha_1^{(1)}=\frac{l'+1}{2m}$ and $a=m_0\omega^2/2$.

Noncommutative Phase-space Hamiltonian for (MCMpH)

Formalism of Boopp's shift

Based on the previous works [31-43], we give a brief review to the fundamental principles of modified Schrödinger equation in (NC-3D: RSP), to achieve this goal we apply the important 4-steps on the ordinary (SE):

- We replace ordinary three dimensional Hamiltonian operators $\hat{H}(p_i,x_i)$ by noncommutative new Hamiltonian operator $\hat{H}(\hat{p}_i, \hat{x}_i)$,

• We replace ordinary complex wave function $\Psi(\vec{r})$ by new complex wave function $\hat{\Psi}(\vec{\hat{r}})$,

• We replace ordinary energy $E_{n,l}$ by noncommutative energy E_{nc},

• The forth step correspond to replace the ordinary old product by new star product.

Which allow us to constructing the modified (SE) in both (NC-3D: RSP) as:

$$\hat{H}(\hat{p}_i, \hat{x}_i) * \hat{\Psi}(\vec{\hat{r}}) = E_{nc-qc} \hat{\Psi}(\vec{\hat{r}}) \quad (12)$$

The Boopp's shift method allows finding the reduced following (SE) without star product:

$$\hat{H}(\hat{p}_i, \hat{x}_i) \psi(\vec{r}) = E_{nc} \psi(\vec{r}) \quad (13)$$

Where the modified Hamiltonian $H(\hat{p}_i, \hat{x}_i)$ defined as a function of two operators $\hat{x}_i$ and $\hat{p}_i$:

$$H_{nc-3is}(\hat{p}_i, \hat{x}_i) = \frac{\hat{p}_i^2}{2m_0} + V_{nc}(\hat{r}) \quad (14)$$

And the modified three dimensional potential $V_{nc}(\hat{r})$ obtained by the following procedure:

$$V_{nc}(\hat{r}) = a\hat{r}^2 + b\hat{r} - \frac{C}{\hat{r}} - \frac{d}{\hat{r}^2} \quad (15)$$

The two $\hat{x}_i$ and $\hat{p}_i$ operators in (NC-3D: RSP) are given by [31-43]:

$$\hat{x}_i = x_i - \frac{\theta_{ij}}{2} p_j \text{ and } \hat{p}_i = p_i - \frac{\overline{\theta}_{ij}}{2} x_j \quad (16)$$

On based to our references [37-40], we can write the two operators $\hat{r}^2$ and $\hat{p}^2$ in noncommutative three dimensional spaces and phases as follows:

$$\hat{r}^2 = r^2 - \vec{\mathbf{L}}\vec{\Theta}$$
$$\frac{\hat{p}^2}{2m_0} = \frac{p^2}{2m_0} + \frac{\vec{\mathbf{L}}\vec{\overline{\boldsymbol{\theta}}}}{2m_0} \quad (17)$$

Where the two couplings $\mathbf{L}\Theta$ and $\vec{\mathbf{L}}\vec{\overline{\boldsymbol{\theta}}}$ are given by:

$$\mathbf{L}\Theta \equiv L_x\Theta_{12} + L_y\Theta_{23} + L_z\Theta_{13} \quad (17.1)$$

and

$$\vec{\mathbf{L}}\vec{\overline{\theta}} \equiv L_x\overline{\theta}_{12} + L_y\overline{\theta}_{23} + L_z\overline{\theta}_{13} \quad (17.2)$$

With $\Theta \equiv \frac{\boldsymbol{\theta}}{2}$. After straightforward calculations one can obtains the different terms in (NC-3D: RSP) as follows:

$$a\hat{r}^2 = ar^2 - a\vec{\mathbf{L}}\vec{\Theta}$$
$$b\hat{r} = br - \frac{b}{2r}\vec{\mathbf{L}}\vec{\Theta}$$
$$\frac{c}{\hat{r}} = \frac{c}{r} + \frac{c}{2r^3}\vec{\mathbf{L}}\vec{\Theta} \quad (18)$$
$$\frac{d}{\hat{r}^2} = \frac{d}{r^2} + \frac{d}{r^4}\vec{\mathbf{L}}\vec{\Theta}$$

Which allow us to writing the modified three dimensional studied potential (MCMpH) in (NC-3D: RSP) as follows:

$$V_{nc-mcph}(\hat{r}) = ar^2 + br - \frac{c}{r} - \frac{d}{r^2} + V_{pert}(r, \Theta, \overline{\theta}) \quad (19)$$

It is clear that, the first 4-terms in eq. (19) represent the ordinary potential while the rest term is produced by the deformation of space and phase. The global perturbative potential operators $V_{pert-mcph}(r, \Theta, \overline{\theta})$ for studied potential (MCMpH) in both (NC-3D: RSP) will be written as:

$$V_{pert-mcph}(r, \Theta, \overline{\theta}) = \left(\frac{d}{r^4} + \frac{c}{2r^3} - \frac{b}{2r} - a\right)\vec{\mathbf{L}}\vec{\Theta} + \frac{1}{2m_0}\vec{\mathbf{L}}\vec{\overline{\boldsymbol{\theta}}} \quad (20)$$

The Spin-orbital Noncommutative Hamiltonian for (CMpH) in (NC: 3D- RSP)

In order, to discover the new contribution of (MCMpH), we replace the two couplings $\vec{\mathbf{L}}\vec{\Theta}$ and $\vec{\mathbf{L}}\vec{\overline{\boldsymbol{\theta}}}$ by $\alpha\Theta\overleftrightarrow{S}\overleftrightarrow{L}$ and $\alpha\overline{\theta}\overleftrightarrow{S}\overleftrightarrow{L}$, respectively, then the above perturbed operator becomes as:

$$H_{\text{pert-mcph}}(r, \Theta, \overline{\theta}) = \alpha\left\{\Theta\left(\frac{d}{r^4} + \frac{c}{2r^3} - \frac{b}{2r} - a\right) + \frac{\overline{\theta}}{2m_0}\right\}\vec{\mathbf{L}}\vec{S} \quad (21)$$

Here $\overleftrightarrow{S}$ denote the spin of a fermionic particle (quark or electron). Now, we replace the spin-orbital interaction $\vec{\mathbf{L}}\vec{S}$ by $G^2 = \frac{1}{2}\left(\overleftrightarrow{J}^2 - \overleftrightarrow{L}^2 - \overleftrightarrow{S}^2\right)$ to obtain the new physical form of the eq. (21) as:

$$H_{\text{pert-mcph}}(r, \Theta, \overline{\theta}) = \left\{\alpha\Theta\left(\frac{3c}{r^8} + \frac{2b}{r^6} - a\right) + \frac{\overline{\theta}}{2m_0}\right\}G^2 \quad (22)$$

As is well known, ($\overleftrightarrow{J}^2$, $\overleftrightarrow{L}^2$, $\overleftrightarrow{S}^2$ and s_z) formed complete basis on quantum mechanics, then the operator $\left(\overleftrightarrow{J}^2 - \overleftrightarrow{L}^2 - \overleftrightarrow{S}^2\right)$ will be gives 2-eigenvalues $k_{\mp} \equiv \frac{1}{2}\left\{\left(l \pm \frac{1}{2}\right)(l + \frac{1}{2} \pm 1) + l(l+1) - \frac{3}{4}\right\}$, corresponding $j = l \pm \frac{1}{2}$ respectively [31-43]. Then, one can form a diagonal matrix $\hat{H}_{so-cmph}$ of order (3×3), with non-null elements $(\hat{H}_{so-cmph})_{11}$, $(\hat{H}_{so-cmph})_{22}$ and $(\hat{H}_{so-cmph})_{33}$ for (MCMpH) potential in both (NC-3D: RSP):

$$(\hat{H}_{so-cmph})_{11} = k_{+}\alpha\left\{\Theta\left(\frac{d}{r^4} + \frac{c}{2r^3} - \frac{b}{2r} - a\right) + \frac{\overline{\theta}}{2m_0}\right\} \text{if } j = l + \tfrac{1}{2} \Rightarrow \text{spin up}$$
$$(\hat{H}_{so-cmph})_{22} = k_{-}\alpha\left\{\Theta\left(\frac{d}{r^4} + \frac{c}{2r^3} - \frac{b}{2r} - a\right) + \frac{\overline{\theta}}{2m_0}\right\} \text{if } j = l - \tfrac{1}{2} \Rightarrow \text{spin down} \quad (23)$$
$$(\hat{H}_{so-cmph})_{33} = 0$$

After profound straightforward calculation, one can show that, the two radial functions $\psi_{n,l}(r)$ and $\phi_{n,l}(r)$ satisfied the following differential equations, in (NC-3D: RSP) for (MCMpH):

$$\left(\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}-\frac{l(l+1)}{r^2}\right)\psi_{n,l}(r)+2\mathrm{m}_0\begin{bmatrix}E_{n,l}-ar^2-br+\frac{c}{r}+\frac{d}{r^2}\\ -\alpha\left\{\Theta\left(\frac{d}{r^4}+\frac{c}{2r^3}-\frac{b}{2r}-a\right)+\frac{\bar{\theta}}{2m_0}\right\}\vec{\mathbf{L}}\vec{S}\end{bmatrix}\psi_{n,l}(r)=0 \quad \ldots(24.1)$$

and

$$\frac{d^2\varphi_{n,l}(r)}{dr^2}+\begin{bmatrix}\varepsilon_{n,l}-a_1r^2-b_1r+\frac{c_1}{r}+\frac{d}{r^2}+\frac{d_1-l(l+1)}{r^2}\\ -\alpha\left\{\Theta\left(\frac{d}{r^4}+\frac{c}{2r^3}-\frac{b}{2r}-a\right)+\frac{\bar{\theta}}{2m_0}\right\}\vec{\mathbf{L}}\vec{S}\end{bmatrix}\varphi_{n,l}(r)=0 \quad \ldots(24.2)$$

The Exact Spectrum of Ground States Produced by Noncommutative Spin-orbital Hamiltonian $\hat{H}_{so-cmph}$ for (MCMpH) in (NC: 3D- RSP)

Now, the aim of this subsection is to obtain the modifications to the energy levels for ground states $E_{\text{u-cmph}}$ and $E_{\text{d-cmph}}$ for spin up and spin down, respectively, at first order of two parameters Θ and $\bar{\theta}$. In order to achieve this goal, we apply the standard perturbation theory:

$$E_{\text{u-cmph}}=\alpha|N_{0l}|^2k_+\int_0^{+\infty}r^{(l'-1)+2}\exp\left(-\beta r^2-\gamma r\right)\left(\Theta\left(\frac{d}{r^4}+\frac{c}{2r^3}-\frac{b}{2r}-a\right)+\frac{\bar{\theta}}{2m_0}\right)dr \quad ..(25.1)$$

$$E_{\text{d-cmph}}=\alpha|N_{0l}|^2k_-\int_0^{+\infty}r^{(l'-1)+2}\exp\left(-\beta r^2-\gamma r\right)\left(\Theta\left(\frac{d}{r^4}+\frac{c}{2r^3}-\frac{b}{2r}-a\right)+\frac{\bar{\theta}}{2m_0}\right)dr \quad ..(25.2)$$

Where, $\beta=\sqrt{2m_0a}$ and $\gamma=\frac{4m_0c}{l'+1}$, a direct simplification gives:

$$E_{\text{u-cmph}}=\alpha|N_{0l}|^2k_+\left\{\theta\sum_{i=1}^{4}T_i+\frac{\bar{\theta}}{2m_0}T_5\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots(26.1)$$

$$E_{\text{d-cmph}}=\alpha|N_{0l}|^2k_-\left\{\theta\sum_{i=1}^{4}T_i+\frac{\bar{\theta}}{2m_0}T_5\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots(26.2)$$

Where, the five terms $T_i\left(i=\overline{1,5}\right)$ are given by:

$$T_1=d\int_0^{+\infty}r^{(l'-2)-1}\exp\left(-\beta r^2-\gamma r\right)dr,$$

$$T_2=\frac{c}{2}\int_0^{+\infty}r^{(l'-1)-1}\exp\left(-\beta r^2-\gamma r\right)dr,$$

$$T_3=-\frac{b}{2}\int_0^{+\infty}r^{(l'+1)-1}\exp\left(-\beta r^2-\gamma r\right)dr, \quad \ldots\ldots\ldots\ldots\ldots(27)$$

$$T_4=-aT_5=-a\int_0^{+\infty}r^{(l'+2)-1}\exp\left(-\beta r^2-\gamma r\right)dr$$

In order to obtain the above integrals, we applying the following special integration [50]:

$$\int_0^{+\infty}x^{\nu-1}\exp\left(-\beta x^2-\gamma x\right)dx=(2\beta)^{-\frac{\nu}{2}}\Gamma(\nu)D_{-\nu}\left(\frac{\gamma}{\sqrt{2\beta}}\right) \quad \ldots(28)$$

Where $D_{-\nu}\left(\frac{\gamma}{\sqrt{2\beta}}\right)$ denote to the Parabolic cylinder functions function, $\Gamma(\nu)$ Gamma function $\mathrm{Re}l(\beta)\rangle 0$ and $\mathrm{Re}l(\nu\rangle 0)$. After straightforward calculations, we can obtain the explicitly results:

$$T_1=d(2\beta)^{-\frac{l'-2}{2}}\Gamma(l'-2)D_{-(l'-2)}\left(\frac{\gamma}{\sqrt{2\beta}}\right),$$

$$T_2=\frac{c}{2}(2\beta)^{-\frac{l'-1}{2}}\Gamma(l'-1)D_{-(l'-1)}\left(\frac{\gamma}{\sqrt{2\beta}}\right), \quad \ldots\ldots\ldots\ldots\ldots\ldots(29)$$

$$T_3=-\frac{b}{2}(2\beta)^{-\frac{l'+1}{2}}\Gamma(l'+1)D_{-(l'+1)}\left(\frac{\gamma}{\sqrt{2\beta}}\right),$$

$$T_4=-aT_5=-a(2\beta)^{-\frac{l'+2}{2}}\Gamma(l'+2)D_{-(l'+2)}\left(\frac{\gamma}{\sqrt{2\beta}}\right)$$

Inserting the above expressions into equations (26.1) and (26.2), one obtains the following results for exact modifications of ground states E_{u} and E_{d} produced by new spin-orbital effect for (MCMPH):

$$E_{\text{u-cmph}}=\alpha\frac{(2\quad)^{'/2}\exp\left(\frac{m\ c}{2\ (\ '\ 1)}\right)}{(l\ )D\left(\frac{m\ c}{l'\ 1}\sqrt{\frac{}{2m}}\right)}k_+\left\{\theta T_{nc-0s}+\bar{\theta}T_{nc-0p}\right\} \quad (30.1)$$

$$E_{\text{d-cmph}}=\alpha\frac{(2m_0\omega)^{l'/2}\exp\left(-\frac{m_0c^2}{2\omega(l'+1)^2}\right)}{\Gamma(l')D_{-l'}\left(\frac{4m_0c}{l'+1}\sqrt{\frac{1}{2m_0\omega}}\right)}k_-\left\{\theta T_{nc-0s}+\bar{\theta}T_{nc-0p}\right\} \quad (30.2)$$

Where the two factors T_{nc-0} and T_{nc-0p} are given by:

$$T_{nc-0s}\equiv d(2\beta)^{-\frac{l'-2}{2}}\Gamma(l'-2)D_{-(l'-2)}\left(\frac{\gamma}{\sqrt{2\beta}}\right)+\frac{c}{2}(2\beta)^{-\frac{l'-1}{2}}\Gamma(l'-1)D_{-(l'-1)}\left(\frac{\gamma}{\sqrt{2\beta}}\right) \quad (31.1)$$
$$-\frac{b}{2}(2\beta)^{-\frac{l'+1}{2}}\Gamma(l'+1)D_{-(l'+1)}\left(\frac{\gamma}{\sqrt{2\beta}}\right)-a(2\beta)^{-\frac{l'+2}{2}}\Gamma(l'+2)D_{-(l'+2)}\left(\frac{\gamma}{\sqrt{2\beta}}\right)$$

$$T_{nc-0pis}\equiv(2\beta)^{-\frac{l'+2}{2}}\Gamma(l'+2)D_{-(l'+2)}\left(\frac{\gamma}{\sqrt{2\beta}}\right) \quad \ldots\ldots\ldots\ldots(31.2)$$

It's important to notice that the above two terms T_{nc-0s} and T_{nc-0p} are represent the noncommutative geometry of space and phase, respectively.

The Exact Spectrum of First Excited States Produced by Noncommutative Spin-orbital Hamiltonian $\hat{H}_{so-cmph}$ for (MCMpH) in (NC: 3D- RSP)

The aim of this subsection is to obtain the new modifications to the energy levels for first excited states E_{u1} and E_{d1} corresponding spin up and spin down, respectively at first order of two parameters Θ and θ for (MCMpH) which are obtained by applying the standard perturbation theory as:

$$E_{\text{u--cmph1}} = \alpha \left| N_1 \right| \ k \int^{+\infty} \left(r^{(l'-1)/+4} + \left(\alpha_1^{(1)}\right) \ r^{(l'-1)/+2} - \ \alpha_1^{(1)} r^{(l'-1)/+3} \right) \exp\left(-\alpha' r \ -\beta' b r\right)\left(\Theta\left(\frac{d}{r^4}+\frac{c}{2r^3}-\frac{b}{2r}-a\right)+\frac{-}{2m}\right)dr \quad (32.1)$$

$$E_{\text{d--cmph1}} = \alpha \left| N_{1l} \right|^2 k_- \int_0^{+\infty} \left(r^{(l'-1)/+4} + \left(\alpha_1^{(1)}\right)^2 r^{(l'-1)/+2} - 2\alpha_1^{(1)} r^{(l'-1)/+3} \right) \exp\left(-\alpha' r^2 -\beta' b r\right)\left(\Theta\left(\frac{d}{r^4}+\frac{c}{2r^3}-\frac{b}{2r}-a\right)+\frac{\overline{\theta}}{2m_0}\right)dr \quad (32.2)$$

Where, (α', β') are equals $(\sqrt{2m_0 a}, 2\sqrt{\frac{m_0}{2a}}b)$ and then a direct simplification to the above equations (32.1) and (32.2) gives:

$$E_{\text{u--cmph1}} = \alpha \left| N_{1l} \right|^2 k_+ \left\{ \Theta \sum_{i=1}^{12} L_i + \frac{\overline{\theta}}{2m_0} \sum_{i=13}^{15} T_i \right\} \quad \text{..............(33)}$$

$$E_{\text{d--cmph1}} = \alpha \left| N_{1l} \right|^2 k_- \left\{ \Theta \sum_{i=1}^{12} L_i + \frac{\overline{\theta}}{2m_0} \sum_{i=13}^{15} T_i \right\} \quad \text{.............(34)}$$

Where, the 15- terms $L_i\left(i=\overline{1,15}\right)$ are given by:

$$L_1 = d \int_0^{+\infty} r^{l'-1} \exp\left(-\alpha' r^2 - \beta' b r\right) dr,$$

$$L_2 = \frac{c}{2} \int_0^{+\infty} r^{(l'+1)/-1} \exp\left(-\alpha' r^2 - \beta' b r\right) dr \quad \text{...........(35)}$$

$$L_3 = -\frac{b}{2} \int_0^{+\infty} r^{(l'+3)/-1} \exp\left(-\alpha' r^2 - \beta' b r\right) dr,$$

$$L_4 = -a \int_0^{+\infty} r^{(l'+4)/-1} \exp\left(-\alpha' r^2 - \beta' b r\right) dr$$

$$L_5 = \left(\alpha_1^{(1)}\right)^2 d \int_0^{+\infty} r^{l'-2/-1} \exp\left(-\alpha' r^2 - \beta' b r\right) dr,$$

$$L_6 = \left(\alpha_1^{(1)}\right)^2 \frac{c}{2} \int_0^{+\infty} r^{(l'-1)/-1} \exp\left(-\alpha' r^2 - \beta' b r\right) dr \quad \text{.......(36.1)}$$

$$L_7 = -\left(\alpha_1^{(1)}\right)^2 \frac{b}{2} \int_0^{+\infty} r^{(l'+1)/-1} \exp\left(-\alpha' r^2 - \beta' b r\right) dr,$$

$$L_8 = -a \left(\alpha_1^{(1)}\right)^2 \int_0^{+\infty} r^{(l'+2)/-1} \exp\left(-\alpha' r^2 - \beta' b r\right) dr$$

$$L_9 = -2\alpha_1^{(1)} d \int_0^{+\infty} r^{(l'-1)-1} \exp\left(-\alpha' r^2 - \beta' b r\right) dr,$$

$$L_{10} = -\alpha_1^{(1)} c \int_0^{+\infty} r^{l'-1} \exp\left(-\alpha' r^2 - \beta' b r\right) dr \quad \text{...............(36.2)}$$

$$L_{11} = \alpha_1^{(1)} b \int_0^{+\infty} r^{(l'+2)/-1} \exp\left(-\alpha' r^2 - \beta' b r\right) dr,$$

$$L_{12} = 2a\alpha_1^{(1)} \int_0^{+\infty} r^{(l'+3)/-1} \exp\left(-\alpha' r^2 - \beta' b r\right) dr$$

$$L_{13} = \int_0^{+\infty} r^{l'+4-1} \exp\left(-\alpha' r^2 - \beta' b r\right) dr,$$

$$L_{14} = \left(\alpha_1^{(1)}\right)^2 \int_0^{+\infty} r^{(l'+2)/-1} \exp\left(-\alpha' r^2 - \beta' b r\right) dr \quad \text{...........(36.3)}$$

$$L_{15} = -2\alpha_1^{(1)} \int_0^{+\infty} r^{(l'+3)/-1} \exp\left(-\alpha' r^2 - \beta' b r\right) dr$$

In order to obtain the results of above equations, we apply the special integral which represents by eq. (28):

$$L_1 = d(2\beta')^{-\frac{l'}{2}} \Gamma(l') D_{-l'}\left(\frac{\gamma'}{\sqrt{2\beta'}}\right),$$

$$L_2 = \frac{c}{2}(2\beta')^{-\frac{l'+1}{2}} \Gamma(l'+1) D_{-(l'+)1}\left(\frac{\gamma'}{\sqrt{2\beta'}}\right) \quad \text{......................... (37.1)}$$

$$L_3 = -\frac{b}{2}(2\beta')^{-\frac{'+3}{2}} \Gamma(l'+3) D_{-(l'+)3}\left(\frac{\gamma'}{\sqrt{2\beta'}}\right),$$

$$L_4 = -a(2\beta')^{-\frac{l'+4}{2}} \Gamma(l'+4) D_{-(l'+4)}\left(\frac{\gamma'}{\sqrt{2\beta'}}\right)$$

$$L_5 = \left(\alpha_1^{(1)}\right)^2 d(2\beta')^{-\frac{l'-2}{2}} \Gamma(l'-2) D_{-(l'-2)}\left(\frac{\gamma'}{\sqrt{2\beta'}}\right)$$

$$L_6 = \left(\alpha_1^{(1)}\right)^2 \frac{c}{2}(2\beta)^{-\frac{v}{2}l'-1} \Gamma(l'-1) D_{-(l'-)1}\left(\frac{\gamma}{\sqrt{2\beta}}\right) \quad \text{...............(37.2)}$$

$$L_7 = -\left(\alpha_1^{(1)}\right)^2 \frac{b}{2}(2\beta')^{-\frac{l'+1}{2}} \Gamma(l'+1) D_{-(l'+)1}\left(\frac{\gamma'}{\sqrt{2\beta'}}\right)$$

$$L_8 = -a\left(\alpha_1^{(1)}\right)^2 (2\beta')^{-\frac{l'+2}{2}} \Gamma(l'+2) D_{-(l'+2)}\left(\frac{\gamma'}{\sqrt{2\beta'}}\right)$$

$$L_9 = -2\alpha_1^{(1)} d\,(2\beta')^{-\frac{l'-1}{2}} \Gamma(l'-1) D_{-(l'-1)}\left(\frac{\gamma'}{\sqrt{2\beta}}\right),$$

$$L_{10} = -\alpha_1^{(1)} c\,(2\beta)^{-\frac{l'}{2}} \Gamma(l') D_{-l'}\left(\frac{\gamma'}{\sqrt{2\beta}}\right) \quad \text{...................(37.3)}$$

$$L_{11} = \alpha_1^{(1)} b\,(2\beta)^{-\frac{l'+2}{2}} \Gamma(l'+2) D_{-(l'+2)}\left(\frac{\gamma'}{\sqrt{2\beta}}\right)$$

$$L_{12} = 2a\alpha_1^{(1)} (2\beta)^{-\frac{l'+3}{2}} \Gamma(l'+3) D_{-(l'+)3}\left(\frac{\gamma'}{\sqrt{2\beta}}\right)$$

$$L_{13} = (2\beta)^{-\frac{l'+4}{2}} \Gamma(l'+4) D_{-(l'+4)}\left(\frac{\gamma}{\sqrt{2\beta}}\right),$$

$$L_{14} = \left(\alpha_1^{(1)}\right)^2 (2\beta)^{-\frac{l'+2}{2}} \Gamma(l'+2) D_{-(l'+2)}\left(\frac{\gamma'}{\sqrt{2\beta}}\right) \quad \text{..........(37.4)}$$

$$L_{15} = -2\alpha_1^{(1)} (2\beta)^{-\frac{l'+3}{2}} \Gamma(l'+2) D_{-(l'+3)}\left(\frac{\gamma'}{\sqrt{2\beta}}\right)$$

The explicit results obtained above allow us to get the exact modifications $E_{\text{u-cmph1}}$ and $E_{\text{d-cmph1}}$ of degenerated first excited states corresponding two polarized states produced by new spin-orbital Hamiltonian operator $\hat{H}_{so-cmph}$:

$$E_{\text{u--cmph1}} = \frac{\alpha(2m_0\omega)^{l'/2} \exp\left(-\frac{b^2}{2m_0\omega^3}\right)}{\frac{1}{2m_0\omega}\Gamma(l'+2)S_1 + \alpha_1^{(1)2}\Gamma(l')S_2 - 2\alpha_1^{(1)}\sqrt{\frac{1}{2m_0}}\Gamma(l'+1)S_3} k_+ \left\{\begin{matrix} \mathrm{O}L_{nc-1s} \\ +\frac{\overline{\theta}}{2m_0} L_{nc-1p} \end{matrix}\right\} \quad \text{....(38.1)}$$

and

$$E_{\text{d--cmph1}} = \frac{\alpha(2m_0\omega)^{l'/2} \exp\left(-\frac{b^2}{2m_0\omega^3}\right)}{\frac{1}{2m_0\omega}\Gamma(l'+2)S_1 + \alpha_1^{(1)2}\Gamma(l')S_2 - 2\alpha_1^{(1)}\sqrt{\frac{1}{2m_0}}\Gamma(l'+1)S_3} k_- \left\{\begin{matrix} \mathrm{O}L_{nc-1s} \\ +\frac{\overline{\theta}}{2m_0} L_{nc-1p} \end{matrix}\right\} \quad \text{....(38.2)}$$

Where the two factors L_{nc-1} and L_{nc-1p} are given by the following form, respectively:

$$L_{nc-1s} \equiv \sum_{i=1}^{12} L_i \quad \text{and} \quad L_{nc-1p} \equiv \sum_{i=13}^{15} L_i \quad \text{....................(39)}$$

The Exact Spectrum Produced by Noncommutative Magnetic Hamiltonian $\hat{H}_{m-cmph}$ for (MCMpH) in (NC: 3D- RSP)

On the other hand, it is possible to found another automatically symmetry for (CMpH) related to the influence of an external uniform magnetic field, generated from the effect of the new geometry of space and phase, it is deduced by the two following two replacements:

$$\Theta \to \chi B \quad \text{and} \quad \overline{\theta} \to \overline{\sigma} B \text{ .... (40)}$$

Here χ and $\overline{\sigma}$ are infinitesimal real two proportional's constants and to simplify the calculations we choose the magnetic field $\overleftrightarrow{B} = B\overleftrightarrow{k}$ and then we can make the following translation:

$$\left\{\Theta\left(\frac{d}{r^4}+\frac{c}{2r^3}-\frac{b}{2r}-a\right)+\frac{\overline{\theta}}{2m_0}\right\}\vec{B}\vec{L} \to \left(\chi\left(\frac{d}{r^4}+\frac{c}{2r^3}-\frac{b}{2r}-a\right)+\frac{\overline{\sigma}}{2m_0}\right) \text{ ...... (41)}$$

Which allow us to introduce the modified new magnetic Hamiltonian $\hat{H}_{m-cmph}$ in (NC-3D: RSP) for (MCMpH) as:

$$\hat{H}_{m-cmph} = \left(\chi\left(\frac{d}{r^4}+\frac{c}{2r^3}-\frac{b}{2r}-a\right)+\frac{\overline{\sigma}}{2m_0}\right)\left(\vec{B}\vec{J}+\hat{H}_z\right) \text{ ...... (42)}$$

Here $\hat{H}_z \equiv -\overleftrightarrow{S}\overleftrightarrow{B}$ denote to the ordinary operator of Hamiltonian for of Zeeman Effect in quantum mechanics. To obtain the exact noncommutative magnetic modifications of energy ($E_{\text{mag-0}}$, $E_{\text{mag-1}}$) for (MCMpH) we just replaced the 3-parameters: k_+, Θ and $\overline{\theta}$ in the Eqs.(30.1) and (38.1) by the following new parameters: m with ($-l \le m \le +l$), χ and $\overline{\sigma}$, respectively:

$$E_{\text{mag-0}} = \alpha \frac{(2m_0\omega)^{l'/2} \exp\left(-\frac{m_0 c^2}{2\omega(l'+1)^2}\right)}{\Gamma(l')D_{-l'}\left(\frac{4m_0 c}{l'+1}\sqrt{\frac{1}{2m_0\omega}}\right)} Bm\left\{\chi T_{nc-0s} + \frac{\overline{\sigma}T_{nc-0p}}{2m_0}\right\} \text{ ...(43.1)}$$

$$E_{\text{mag-1}} = \frac{\alpha(2m_0\omega)^{l'/2} \exp\left(-\frac{b^2}{2m_0\omega^3}\right)}{\frac{1}{2m_0\omega}\Gamma(l'+2)S_1 + \alpha_1^{(1)2}\Gamma(l')S_2 - 2\alpha_1^{(1)}\sqrt{\frac{1}{2m_0}}\Gamma(l'+1)S_3} Bm\left\{\chi T_{nc-0s} + \frac{\overline{\sigma}T_{nc-0p}}{2\mathrm{m}_0}\right\} \text{...(43.2)}$$

Where $E_{\text{mag-0}}$ and $E_{\text{mag-1}}$ are the exact magnetic modifications of spectrum corresponding the ground states and first excited states, respectively. The new global exact spectrum of lowest excited states for (MCMpH) in (NC-3D: RSP) produced by the diagonal elements of noncommutative Hamiltonian operator $\hat{H}_{nc-cmph}$. It is clearly, that the obtained previous results which are presented by Eqs. (30.1), (30.2), (38.1), (38.2), (43.1) and (43.2) of eigenvalues of energies are reels and then the noncommutative diagonal Hamiltonian operator $\hat{H}_{nc-cmph}$ will be Hermitian operator. Furthermore, we can obtain the explicit physical form of this operator based on the results (23) and (42) for (CMpH), its represent by diagonal, noncommutative matrix of order (3×3), with elements $\left(\hat{H}_{nc-cmph}\right)_{11}$ $\left(\hat{H}_{nc-cmph}\right)_{22}$ and $\left(\hat{H}_{nc-cmph}\right)_{33}$ in both (NC-3D: RSP):

$$\left(\hat{H}_{nc-cmph}\right)_{11} = -\frac{\Delta}{2\mathrm{m}_0} + ar^2 + br - \frac{c}{r} - \frac{d}{r^2} + k_+\left\{\Theta\left(\frac{d}{r^4}+\frac{c}{2r^3}-\frac{b}{2r}-a\right)+\frac{\overline{\theta}}{2m_0}\right\}$$
$$+\left(\chi\left(\frac{d}{r^4}+\frac{c}{2r^3}-\frac{b}{2r}-a\right)+\frac{\overline{\sigma}}{2m_0}\right)BL_z \quad \text{if } j = l+\tfrac{1}{2} \Rightarrow \text{spin up} \quad \text{(44.1)}$$

$$\left(\hat{H}_{nc-cmph}\right)_{22} = -\frac{\Delta}{2\mathrm{m}_0} + ar^2 + br - \frac{c}{r} - \frac{d}{r^2} + k_-\left\{\Theta\left(\frac{d}{r^4}+\frac{c}{2r^3}-\frac{b}{2r}-a\right)+\frac{\overline{\theta}}{2m_0}\right\}$$
$$+\left(\chi\left(\frac{d}{r^4}+\frac{c}{2r^3}-\frac{b}{2r}-a\right)+\frac{\overline{\sigma}}{2m_0}\right)BL_z \quad \text{if } j = l-\tfrac{1}{2} \Rightarrow \text{spin -down} \quad \text{(44.2)}$$

and

$$\left(\hat{H}_{nc-cmph}\right)_{33} = -\frac{\Delta}{2m_0} + ar^2 + br - \frac{c}{r} - \frac{d}{r^2} \text{ ....(44.3)}$$

$$\to \text{Non} - \text{polarised} - \text{electron}$$

Now we find the corresponding global modified energies ($E_{\text{nc u0}}$ - $E_{\text{nc d0}}$) and ($E_{\text{nc u1}}$ - $E_{\text{nc d1}}$) for ground and first excited states of a particle fermionic with spin up and spin moving in the (MCMpH), referring to Eqs. (10.1), (10.2), (30.1), (30.2), (38.1), (38.2), (43.1) and (43.2), we fond the results as follows:

$$E_{\text{nc u0}} = \sqrt{\frac{\quad}{2\mathrm{m}}}(3+l') - \frac{m\ c}{(\ '\ \ 1)} + \frac{m_0(2m_0\quad)^{'/2}\exp\left(\frac{m\ c}{2\ \ (\ '\ \ 1)}\right)}{(l\ )D\ \ \left(\frac{m\ c}{l'\ \ 1}\sqrt{\frac{\quad}{2m}}\right)} \text{ ...(45.1)}$$

$$\left\{k_+\left\{\theta T_{nc-0s} + \frac{\overline{\quad}}{2m_0}T_{nc-0p}\right\} + Bm\left\{\chi T_{nc-0s} + \frac{\overline{\quad}_{nc\quad p}}{2m_0}\right\}\right\}$$

$$E_{\text{nc d0}} = \sqrt{\frac{a}{2m_0}}(3+l') - \frac{2m_0c^2}{(l'+1)^2} + \alpha\frac{(2m_0\omega)^{l'/2}\exp\left(-\frac{m_0c^2}{2\omega(l'+1)^2}\right)}{\Gamma(l')D_{-l'}\left(\frac{4m_0c}{l'+1}\sqrt{\frac{1}{2m_0\omega}}\right)} \quad ...(45.2)$$

$$\left\{k_-\left\{\theta T_{nc-0s} + \frac{\overline{\theta}}{2m_0}T_{nc-0p}\right\} + Bm\left\{\chi T_{nc-0s} + \frac{\overline{\sigma}T_{nc-0p}}{2m_0}\right\}\right\}$$

and

$$E_{\text{nc u1}} = \sqrt{\frac{a}{2m_0}}(4+l') - \frac{b^2}{4a} + \frac{\alpha(2m_0\omega)^{l'/2}\exp\left(-\frac{b^2}{2m_0\omega^3}\right)}{\frac{1}{2m_0\omega}\Gamma(l'+2)S_1 + \alpha_1^{(1)2}\Gamma(l')S_2 - 2\alpha_1^{(1)}\sqrt{\frac{1}{2m_0}}\Gamma(l'+1)S_3} \quad ...(45.3)$$

$$\left\{k_+\left\{\theta L_{nc-1s} + \overline{\theta}L_{nc-1p}\right\} + Bm\left\{\chi T_{nc-1s} + \frac{\overline{\sigma}T_{nc-1p}}{2m_0}\right\}\right\}$$

and

$$E_{\text{nc d1}} = \sqrt{\frac{a}{2m_0}}(4+l') - \frac{b^2}{4a} + \frac{\alpha(2\mathrm{m}_0\omega)^{l'/2}\exp\left(-\frac{b^2}{2m_0\omega^3}\right)}{\frac{1}{2m_0\omega}\Gamma(l'+2)S_1 + \alpha_1^{(1)2}\Gamma(l')S_2 - 2\alpha_1^{(1)}\sqrt{\frac{1}{2m_0}}\Gamma(l'+1)S_3} \quad ...(45.4)$$

$$\left\{k_-\left\{\theta L_{nc-1s} + \overline{\theta}L_{nc-1p}\right\} + Bm\left\{\chi T_{nc-1s} + \frac{\overline{\sigma}T_{nc-1p}}{2m_0}\right\}\right\}$$

As it's mentioned in our previous works [31-43], the atomic quantum number m can be taken $(2l+1)$ values and we have also two values for $j = l \pm \frac{1}{2}$, thus every state in usually three dimensional space of (MCMpH) will be in (NC-3D: RSP): $2(2l+1)$ sub-states.

It is important to notice that the appearance of the polarization states of a fermionic particle indicates the validity of the results in the field of high energy where Dirac equation is applied, which allowing to the validity to results of present search on the Plank's and Nano scales level. If we make the limits $(\theta_k, \overline{\theta}_k) \rightarrow (0,0)$ we can obtain all the results of ordinary quantum mechanics.

Conclusion

In this article, we have investigated the solutions of the Schrödinger equation for modified (MCMpH) potential. We showed that the obtained degenerated spectrum for the modified studied potential depended by new discrete atomic quantum numbers: $j = l \pm \frac{1}{2}$ and m of electron, our obtained results could be extended to applied at and Plank's and Nano scales.

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